Math 143
Fall, 2006 Name: **Solutions**
Quiz 14 - §8.8

Complete the following problems by deciding if the following integrals converge or diverge. If they converge, find their integral (if possible). Show all work to receive full credit.

1. $\displaystyle\int_0^\infty \frac{16\tan^{-1}x}{1+x^2}\,dx$

$$\begin{aligned}
\int_0^\infty \frac{16\tan^{-1}x}{1+x^2}\,dx &= \lim_{t\to\infty}\int_0^t \frac{16\tan^{-1}x}{1+x^2}\,dx \\
u &= \tan^{-1}x \\
du &= \frac{1}{1+x^2}\,dx \\
&= \lim_{t\to\infty}\int_{x=0}^{x=t} 16u\,du \\
&= \lim_{t\to\infty}\left(8u^2\right|_{x=0}^{x=t} \\
&= \lim_{t\to\infty}\left(8\left(\tan^{-1}x\right)^2\,\right|_{x=0}^{x=t} \\
&= \lim_{t\to\infty}\left(8\left(\tan^{-1}t\right)^2 - 8\left(\tan^{-1}0\right)^2\right) \\
&= 8\left(\frac{\pi}{2}\right)^2 - 8(0)^2 \\
&= 8\frac{\pi^2}{4} - 0 \\
&= 2\pi^2,\ \text{so it converges}
\end{aligned}$$

2. $\displaystyle\int_1^3 \frac{1}{(2x-1)^{\frac{3}{2}}}\,dx$

$$\begin{aligned}
\int_1^3 \frac{1}{(2x-1)^{\frac{3}{2}}}\,dx &= \left(-(2x-1)^{-\frac{1}{2}}\right|_1^3 \\
&= -\frac{1}{\sqrt{2(3)-1}} + \frac{1}{\sqrt{2(1)-1}} \\
&= -\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{1}} \\
&= -\frac{1}{\sqrt{5}} + 1
\end{aligned}$$

3. $\displaystyle\int_{\pi}^{\infty} \frac{2+\cos x}{x}\, dx$

$$\begin{gathered}
-1 \le \cos x \le 1 \\
2-1 \le 2+\cos x \le 1+2 \\
1 \le 2+\cos x \le 3 \\
\frac{1}{x} \le \frac{2+\cos x}{x} \le \frac{3}{x}
\end{gathered}$$

$$\begin{aligned}
\int_{\pi}^{\infty} \frac{2+\cos x}{x} &\ge \int_{\pi}^{\infty} \frac{1}{x}\, dx \\
&= \text{diverges}
\end{aligned}$$

Therefore, $\displaystyle\int_{\pi}^{\infty} \frac{2+\cos x}{x}\, dx$ diverges also.

4. What was Monday's word of the day?

   Clown