Math 143
Fall, 2005
Quiz 14 - §7.7

Name: **Solutions**

Complete the following problems. Show all work to receive full credit.

1. Determine whether the following integrals converge or diverge. If they converge and are integrable, find the integral.

(a) $\int_0^1 (-\ln x)\ dx$

$$= \lim_{t\to 0^+} \int_t^1 -\ln x\ dx$$

$$u = \ln x \qquad v = dx$$

$$du = \frac{1}{x}dx \qquad dv = x$$

$$= \lim_{t\to 0^+} -x\ln x|_t^1 + \int_t^1 dx$$

$$= \lim_{t\to 0^+} -x\ln x + x|_t^1$$

$$= \lim_{t\to 0^+} -\ln 1 + 1 + t\ln t - t$$

$$= 1 + \lim_{t\to 0^+} t\ln t$$

$$= 1 + \lim_{t\to 0^+} \frac{\ln t}{\frac{1}{t}}$$

$$=^{LH} 1 + \lim_{t\to 0^+} \frac{\frac{1}{t}}{\frac{1-}{t^2}}$$

$$= 1 + \lim_{t\to 0^+} t = 1$$

So it converges.

(b) $\displaystyle\int_{\pi}^{\infty} \frac{1+\sin x}{x^2}\ dx$

Since $-1 \leq \sin x \leq 1$, we know that $1 \leq 1 + \sin^x \leq 2$. Therefore, we have

$$\frac{1}{x^2} \leq \frac{1+\sin^2 x}{x^2} \leq \frac{2}{x^2},$$

so we can compare to

$$\int_{pi}^{\infty} \frac{1}{x^2} dx$$

$$= \lim_{t\to\infty} \int_{\pi}^{t} \frac{1}{x^2}\ dx$$

$$= \lim_{t\to\infty} -\frac{1}{x}\Big|_{\pi}^{t}$$

$$= \lim_{t\to\infty} -\frac{1}{t} + \frac{1}{\pi}$$

$$= \frac{1}{\pi}$$

Since this is larger than the desired integral, $\displaystyle\int_{\pi}^{\infty} \frac{1+\sin x}{x^2}\ dx$ converges also.