Math 143
Fall, 2006
Quiz 12 - §8.5

Name: **Solutions**

Complete the following problems by computing the following integrals. Show all work to receive full credit.

1. $\displaystyle\int \frac{\sqrt{x^2-49}}{x}\,dx$

$$x = 7\sec\theta$$

$$dx = 7\sec\theta\tan\theta d\theta$$

$$\begin{aligned}
\int \frac{\sqrt{x^2-49}}{x}\,dx &= \int \frac{\sqrt{49\sec^2\ t-49}}{7\sec\theta}7\sec\theta\tan\theta\ d\theta \\
&= \int \frac{\sqrt{49\tan^2\theta}}{7\sec\theta}7\sec\theta\tan\theta\ d\theta \\
&= \int 7\mid\tan\theta\mid\cdot\tan\theta\ d\theta \\
&= \int 7\tan^2\theta\ d\theta \\
&= 7\int \sec^2\theta - 1\ d\theta \\
&= 7\left(\tan\theta-\theta\right)+C \\
&= 7\left(\frac{\sqrt{x^2-49}}{7}-\sec^{-1}\left(\frac{x}{7}\right)\right)+C \\
&= \sqrt{x^2-49}-7\sec^{-1}\left(\frac{x}{7}\right)+C
\end{aligned}$$

2. $\displaystyle\int \frac{6}{(9x^2+1)^2}\ dx$

$$
\begin{aligned}
\int \frac{6}{(9x^2+1)^2}\ dx &= 6\int \frac{1}{\left(9(x^2+\frac{1}{9})\right)^2}\ dx \\
&= \frac{6}{81}\int \frac{1}{\left(x^2+\frac{1}{9}\right)^2}\ dx \\
&= \frac{6}{81}\int \frac{1}{\left(x^2+\frac{1}{9}\right)^2}\ dx \\
x &= \frac{1}{3}\tan\theta \\
dx &= \frac{1}{3}\sec^2\theta\ d\theta \\
&= \frac{6}{81}\int \frac{1}{\left(\frac{1}{9}\tan^2\theta+\frac{1}{9}\right)^2}\ \frac{1}{3}\sec^2\theta\ d\theta \\
&= \frac{6}{81}\int \frac{1}{\left(\frac{1}{9}\sec^2\theta\right)^2}\cdot\frac{1}{3}\sec^2\theta\ d\theta \\
&= \frac{6}{81}\int \frac{1}{\frac{1}{81}\sec^4\theta}\cdot\frac{1}{3}\sec^2\theta\ d\theta \\
&= 6\cdot\frac{1}{3}\int \frac{1}{\sec^2\theta}\ d\theta \\
&= 2\int \cos^2\theta\ d\theta \\
&= 2\int \frac{1+\cos 2\theta}{2}\ d\theta \\
&= \theta+\frac{1}{2}\sin 2\theta + C \\
&= \theta+\frac{1}{2}2\sin\theta\cos\theta + C \\
&= \tan^{-1}(3x)+\left(\frac{3x}{\sqrt{9x^2+1}}\right)\left(\frac{1}{\sqrt{9x^2+1}}\right)+C \\
&= \tan^{-1}(3x)+\frac{3x}{9x^2+1}+C
\end{aligned}
$$