
Complete the following problems by computing the following integrals. Show all work to receive full credit.

1. $\int_0^{\frac{\pi}{2}} 35 \sin^4 x \cos^3 x \, dx$

$$\begin{aligned} \int_0^{\frac{\pi}{2}} 35 \sin^4 x \cos^3 x \, dx &= 35 \int_0^{\frac{\pi}{2}} \sin^4 x \cos x \cos^2 x \, dx \\ &= 35 \int_0^{\frac{\pi}{2}} \sin^4 x \cos x (1 - \sin^2 x) \, dx \\ &= 35 \int_0^{\frac{\pi}{2}} \sin^4 x \cos x - \sin^6 x \cos x \, dx \\ &= \left. \frac{35}{5} \sin^5 x - \frac{35}{7} \sin^7 x \right|_0^{\frac{\pi}{2}} \\ &= 7 \left(\sin \frac{\pi}{2} \right)^5 - 5 \left(\sin \frac{\pi}{2} \right)^7 - 7 (\sin 0)^5 + 5 (\sin 0)^7 \\ &= 7(1)^5 - 5(1)^7 - 7 \cdot 0 + 5 \cdot 0 \\ &= 7 - 5 = 2 \end{aligned}$$

2. $\int 6 \tan^4 x \, dx$

$$\begin{aligned} \tan^2 x + 1 &= \sec^2 x \\ \tan^2 x &= \sec^2 x - 1 \\ \int 6 \tan^4 x \, dx &= 6 \int \tan^2 x \tan^2 x \, dx \\ &= 6 \int \tan^2 x (\sec^2 x - 1) \, dx \\ &= 6 \int \tan^2 x \sec^2 x - \tan^2 x \, dx \\ &= 6 \cdot \frac{1}{3} \tan^3 x - 6 \int \tan^2 x \, dx \\ &= 2 \tan^3 x - 6 \int \sec^2 x - 1 \, dx \\ &= 2 \tan^3 x - 6 \tan x + 6x + C \end{aligned}$$

$$3. \int \sqrt{\sec^2 x - 1} dx$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\tan^2 x = \sec^2 x - 1$$

$$\begin{aligned} \int \sqrt{\sec^2 x - 1} dx &= \int \sqrt{\tan^2 x} dx \\ &= \int \tan x dx \\ &= \ln |\sec x| + C \end{aligned}$$