
Complete the following problems. Show all work to receive full credit.

1. Evaluate $\int \frac{2}{x^3\sqrt{x^2-1}} dx$

$$\begin{aligned}x &= \sec \theta \\dx &= \sec \theta \tan \theta d\theta \\&= \int \frac{2 \sec \theta \tan \theta d\theta}{\sec^3 \theta \sqrt{\sec^2 \theta - 1}} \\&= \int \frac{1 \sec \theta \tan \theta d\theta}{\sec^3 \theta \tan \theta} \\&= \int \frac{2 d\theta}{\sec^2 \theta} \\&= \int 2 \cos^2 \theta d\theta \\&= \int 2 \left(\frac{1 + \cos 2\theta}{2} \right) d\theta \\&= \int 1 + \cos 2\theta d\theta \\&= \theta + \frac{1}{2} \sin 2\theta + C \\&= \theta + \frac{1}{2} 2 \sin \theta \cos \theta + C \\&= \theta + \sin \theta \cos \theta + C\end{aligned}$$

From the triangle:

$$= \tan^{-1}(\sqrt{x^2-1}) + \frac{\sqrt{x^2-1}}{x} \left(\frac{1}{x}\right) + C$$