

Complete the following problems by computing the following integrals. Show all work to receive full credit.

1.  $\int \frac{8}{x^3(x+2)} dx$

$$\int \frac{8}{x^3(x^2+1)} dx = \int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+1} dx$$

$$Ax^2(x^2+1) + Bx(x^2+1) + C(x^2+1) + (Dx+E)x^3 = 8$$

$$Ax^4 + Ax^2 + Bx^3 + Bx + Cx^2 + C + Dx^4 + Ex^3 = 8$$

$$(A+D)x^4 + (B+E)x^3 + (A+C)x^2 + Bx + C = 8$$

$$A+D=0 \quad B+E=0 \quad A+C=0 \quad B=0 \quad C=8$$

$$D=8 \quad E=0 \quad A=-8$$

$$= \int \frac{-8}{x} + \frac{0}{x^2} + \frac{8}{x^3} + \frac{8x+0}{x^2+1} dx$$

$$= -8 \ln |x| - 4x^{-2} + 4 \ln |x^2+1| + C$$

2.  $\int \frac{1}{x^2-2x+2} dx$

$$\int \frac{1}{x^2-2x+2} dx = \int \frac{1}{(x-2)(x-1)} dx$$

$$= \int \frac{A}{x-2} + \frac{B}{x-1} dx$$

$$A(x-1) + B(x-2) = 1$$

$$A+B=0 \quad -A-2B=1$$

$$A=-B$$

$$B-2B=1$$

$$-B=1$$

$$B=-1$$

$$= \int \frac{1}{x-2} + \frac{-1}{x-1} dx$$

$$= \ln |x-2| - \ln |x-1| + C$$