
Complete the following problems. Show all work to receive full credit.

1. Evaluate $\int \frac{x+3}{2x^3-8x} dx$

$$\begin{aligned} &= \int \frac{x+3}{2x(x^2-4)} dx \\ &= \int \frac{x+3}{2x(x-2)(x+2)} dx \\ &= \int \frac{A}{2x} + \frac{B}{x-2} + \frac{C}{x+2} dx \end{aligned}$$

Therefore, we need to solve:

$$A(x-2)(x+2) + B(2x)(x+2) + C(2x)(x-2) = x+3$$

Plugging in clever numbers we get:

$$\text{If } x = 0 : A(-2)(2) = 3$$

$$-4A = 3$$

$$A = \frac{-3}{4}$$

$$\text{If } x = 2 : B(4)(4) = 5$$

$$16B = 5$$

$$B = \frac{5}{16}$$

$$\text{If } x = -2 : C(-4)(-4) = 1$$

$$16C = 1$$

$$C = \frac{1}{16}$$

Therefore we need to integrate:

$$\begin{aligned} &\int \frac{-3}{4} \frac{1}{2x} + \frac{5}{16} \frac{1}{x-2} + \frac{1}{16} \frac{1}{x+2} dx \\ &= -\frac{3}{4} \cdot \frac{1}{2} \ln |2x| + \frac{5}{16} \ln |x-2| + \frac{1}{16} \ln |x+2| + C \\ &= -\frac{3}{8} \ln |2x| + \frac{5}{16} \ln |x-2| + \frac{1}{16} \ln |x+2| + C \end{aligned}$$