Math 143
Fall, 2005
Quiz 10 - §7.2

Name: **Solutions**

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Complete the following problems. Show all work to receive full credit.

1. Evaluate $\int x^2 \sin 2x \; dx$

$$u = x^2 \qquad dv = \sin 2x \; dx$$
$$du = 2x \; dx \qquad v = -\frac{1}{2}\cos 2x$$

$$= -\frac{1}{2}x^2\cos 2x + \frac{1}{2}\int 2x\cos 2x \; dx$$

$$u = 2x \qquad dv = \cos 2x \; dx$$
$$du = 2 \; dx \qquad v = \frac{1}{2}\sin 2x$$

$$= -\frac{1}{2}x^2\cos 2x + \frac{1}{2}\left(x\sin 2x - \int \sin 2x \; dx\right)$$
$$= -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$$

2. Find the average value of the function $f(t) = 2e^{-t}\cos t$ on the interval $[0, 2\pi]$.

$$f_{avg} = \frac{1}{2\pi}\int_0^{2\pi} 2e^{-t}\cos t \; dt$$
$$= \frac{1}{\pi}\int_0^{2\pi} e^{-t}\cos t \; dt$$

We will just do the integral first:

$$\int e^{-t}\cos t \; dt$$

$$u = e^{-t} \qquad dv = \cos t \; dt$$
$$du = -e^{-t} \; dt \qquad v = \sin t$$
$$= e^{-t}\sin t + \int e^{-t}\sin t \; dt$$
$$u = e^{-t} \qquad dv = \sin t \; dt$$
$$du = -e^{-t} \; dt \qquad v = -\cos t$$
$$= e^{-t}\sin t - e^{-t}\cos t - \int e^{-t}\cos t \; dt$$

So we have

$$2\int e^{-t}\cos t\ dt = e^{-t}\sin t - e^{-t}\cos t$$

$$\int e^{-t}\cos t\ dt = \frac{1}{2}e^{-t}\sin t - \frac{1}{2}e^{-t}\cos t$$

Now using this above:

$$f_{avg} = \frac{1}{\pi}\int_0^{2\pi} e^{-t}\cos t\ dt$$

$$= \frac{1}{\pi}\left(\frac{1}{2}e^{-t}\sin t - \frac{1}{2}e^{-t}\cos t\Big|_0^{2\pi}\right.$$

$$= \frac{1}{\pi}\left(\frac{1}{2}e^{-2\pi}\sin 2\pi - \frac{1}{2}e^{-2\pi}\cos 2\pi - \frac{1}{2}e^{0}\sin 0 + \frac{1}{2}e^{0}\cos 0\right)$$

$$= \frac{1}{\pi}\left(-\frac{1}{2}e^{-2\pi} + \frac{1}{2}\right)$$