

Part I - Definitions and Examples

1. (3 points) State the definition of an integrable function, i.e, f is integrable on $[a, b]$ if:
2. (3 points) State both parts of the Fundamental Theorem of Calculus.
3. (3 points) State the Monotonic Sequence Theorem.
4. (3 points) State the Absolute Convergence Theorem.

Part II - Calculations

5. (6 points) Use a Riemann sum with $n = 4$ and the right endpoint of each subinterval chose to estimate the area under the curve $y = 2x + 1$ on the interval $[0, 4]$. Is this an overestimate or an underestimate? Why?
6. Find the following integrals:

(a) (6 points) $\int x \ln x dx$

(b) (4 points) $\int \frac{\cos(\sin^{-1} x) dx}{\sqrt{1-x^2}}$

(c) (6 points) $\int \frac{2dx}{x^2 - 2x}$

(d) (6 points) $\int \frac{dx}{\sqrt{x^2 - 1}}$

(e) (6 points) $\int_3^5 \frac{1}{x-4} dx$

(f) (6 points) $\int_0^{\infty} \frac{16 \tan^{-1} x dx}{1+x^2}$

7. (5 points) Set up, but **do not integrate** the integral to find the volume of the solid generated by revolving the region bounded by the x -axis, the curve $y = 3x^4$ and the lines $x = 1$ and $x = -1$ about the line $x = 1$.
8. (6 points) Find the area between the curves $y = \cos x$ and $y = \sin x$ between $x = 0$ and $x = \frac{\pi}{4}$.
9. (6 points) Find the first three non-zero terms of the Taylor polynomial for $f(x) = \sqrt{3+x^2}$ near $x = 1$.
10. (3 points) Find the Maclaurin series for e^x . Show all of your work.
11. Determine if the following series converge or diverge. Provide reasons for your answers.

(a) (5 points) $\sum_{n=1}^{\infty} \frac{n^n}{(2^n)^2}$

(b) (5 points) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{3n}\right)^n$

12. (6 points) Find the interval of convergence for the series $\sum_{n=1}^{\infty} \frac{(x-1)^{2n-2}}{(2n-1)!}$

13. Determine if the following series converge absolutely, converge conditionally or diverge. Provide reasons for your answers.

(a) (6 points) $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{\ln n}{n^3}$

(b) (6 points) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2 + 1}{2n^2 + n - 1}$