Math 143
Spring, 2003
Exam 2
Chapter 7 ____

Part I - Definitions and Examples

1. **Part II - Calculations**

2. (12 points) Match the indefinite integral with the correct anti-derivative. Place the correct letter in the blank before the integral:

H	$\int \frac{1}{a^2+u^2}$	A.	$\sin u + C$
K	$\int a^u du$	B.	$-\sin u + C$
I	$\int \csc u \cot u du$	C.	$\ln \mid \cos u \mid + C$
D	$\int \tan u du$	D.	$-\ln \mid \cos u \mid + C$
E	$\int \sec^2 u du$	E.	$\tan u + C$
A	$\int \cos u du$	F.	$\sec u + C$
		G.	$\sin^{-1} \frac{u}{a} + C$
		H.	$\frac{1}{a} \tan^{-1} \frac{u}{a} + C$
		I.	$-\csc u + C$
		J.	$-\cot u + C$
		K.	$\frac{a^u}{\ln a} + C$

3. Find the following integrals:

(a) (10 points) $\int e^x \sin x dx$

$$u = \sin x \qquad dv = e^x dx$$
$$du = \cos x dx \qquad v = e^x$$
$$\int e^x \sin x dx = e^x \sin x - \int e^x \cos x dx$$
$$u = \cos x \qquad dv = e^x dx$$
$$du = -\sin x dx \qquad v = e^x$$
$$\int e^x \sin x dx = e^x \sin x - \left(e^x \cos x + \int e^x \sin x dx \right)$$
$$\int e^x \sin x dx = e^x \sin x - e^x \cos x - \int e^x \sin x dx$$
$$2 \int e^x \sin x dx = e^x \sin x - e^x \cos x$$
$$\int e^x \sin x dx = \frac{1}{2} \left(e^x \sin x - e^x \cos x \right)$$

(b) (6 points) $\int 10^{2\theta} d\theta$

$$= \frac{1}{\ln 10} \frac{1}{2} 10^{2\theta} + C$$
$$= \frac{1}{2 \ln 10} 10^{2\theta} + C$$

(c) (6 points) $\int \sin^3 x \cos x dx$

$$u = \sin x \qquad du = \cos x dx$$
$$= \int u^3 du$$
$$= \frac{1}{4} u^4 + C$$
$$= \frac{1}{4} \sin^4 x + C$$

(d) (12 points) $\int \frac{1}{x^4 + x^2} dx$

$$= \int \frac{1}{x^2 (x^2 + 1)} dx = \int \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 1} dx$$

Therefore,

$$Ax(x^2 + 1) + B(x^2 + 1) + (Cx + D)x^2 = 1$$

So, if $x = 0$, then $B = 1$. Now we multiply out to find the other coefficients:

$$Ax^3 + Ax + x^2 + 1 + Cx^3 + Dx^2 = 1$$
$$(A + C)x^3 + (1 + D)x^2 + Ax + 1 = 1$$

Therefore, $D = -1$, $A = 0$, and $C = 0$. So the integral becomes:

$$\int \frac{0}{x} + \frac{1}{x^2} + \frac{-1}{x^2+1} dx$$

$$= -\frac{1}{x} - \tan^{-1} x + C$$

(e) (6 points) $\int x \tan(x^2) dx$

$$u = x^2 \qquad du = 2xdx$$

$$= \frac{1}{2} \int \tan u du = \frac{1}{2} \ln \mid \sec x^2 \mid + C$$

4. (12 points) Calculate the average value of $f(x) = x^2 e^{4x}$ on the interval $[0, 2]$.

$$f_{avg} = \frac{1}{2} \int_0^2 x^2 e^{4x} dx$$

Do integration by parts:

$$u = x^2 \qquad dv = e^{4x} dx$$

$$du = 2xdx \qquad v = \frac{1}{4} e^{4x}$$

$$= \frac{1}{2} \left(\frac{1}{4} x^2 e^{4x} - \frac{1}{2} \int x e^{4x} dx \right)$$

$$u = x \qquad dv = e^{4x} dx$$

$$du = dx \qquad v = \frac{1}{4} e^{4x} dx$$

$$= \frac{1}{2} \left(\frac{1}{4} x^2 e^{4x} - \frac{1}{2} \left(\frac{1}{4} x e^{4x} - \frac{1}{4} \int e^{4x} dx \right) \right)$$

$$\left(\frac{1}{8} x^2 e^{4x} - \frac{1}{16} x e^{4x} + \frac{1}{16} \frac{1}{4} e^{4x} \mid_0^2 \right.$$

$$= \frac{1}{8} \cdot 4e^8 - \frac{1}{16} \cdot 2e^8 + \frac{1}{64} e^8 - \frac{1}{64}$$

$$= \frac{1}{2} e^8 = \frac{1}{8} e^8 + \frac{1}{64} e^8 - \frac{1}{64}$$

$$= \frac{25}{64} e^8 - \frac{1}{64}$$

5. (6 points) $\int \frac{dx}{\sqrt{x^2 - 1}}$

$$x = \sec \theta$$

$$dx = \sec \theta \tan \theta d\theta$$

$$= \int \frac{\sec \theta \tan \theta}{\tan \theta} d\theta$$

$$= \int \sec \theta d\theta$$

$$= \ln \mid \sec \theta + \tan \theta \mid + C$$

6. (6 points) $\int_3^5 \frac{1}{x-4}dx$

$$= \int_3^4 \frac{1}{x-4}dx + \int_4^5 \frac{1}{x-4}dx$$

Taking these one at a time:

$$\int_3^4 \frac{1}{x-4}dx = \lim_{t\to 4^-} \int_3^t \frac{1}{x-4}dx$$

$$= \lim_{t\to 4^-} \ln \mid x-4 \mid |_3^t$$

$$= \lim_{t\to 4^-} \ln \mid t-4 \mid - \ln 1$$

$$= -\infty$$

Therefore the integral diverges.