Here are some good review questions. The actual exam might be of a different format, but these will help you understand the concepts covered on the exam. Try to do as many of these as you can without looking in your notes or book for guidance.

1. Calculate the following integrals:

   (a) $\int \frac{x-2}{x^2-4x}dx$

   (b) $\int x\sin(x^2)dx$

   (c) $\int x\sin x dx$

   (d) $\int x\cos x dx$

   (e) $\int e^{2x}\sin x dx$

   (f) $\int_1^9 \sqrt{x}\ln x dx$

   (g) $\int \frac{4x^2-7}{2x+3}dx$

   (h) $\int \cos^3\theta d\theta$

   (i) $\int (\csc x - \tan x)^2 dx$

   (j) $\int \sqrt{1-\cos^2\theta}d\theta$

   (k) $\int (\csc x - \sec x)(\sin x + \cos x)dx$

   (l) $\int 2x\sin^{-1}(x^2)dx$

   (m) $\int x\sin^{-1}(x)dx$

   (n) $\int (r^2+r+1)e^r dr$

   (o) $\int \frac{1}{\sqrt{x+1}}dx$

   (p) $\int \frac{6x^3+9x+5}{x^4+3x^2-4}dx$

   (q) $\int \frac{1}{e^{3x}-e^x}dx$ (This one is a little tricky)

   (r) $\int \frac{x^2}{x^3+7}dx$

   (s) $\int_{-1}^2 x^2e^{x^3}dx$

   (t) $\int \sin 3x\cos 5x dx$ (Hint: This requires one of those weird trig. formulas)

   (u) $\int \tan^3 x\sec^3 x dx$

   (v) $\int \sin^3 x\cos x dx$

   (w) $\int \tan^4 x\sec^2 x dx$

2. Calculate these integrals, too:

   (a) $\int x^3\sqrt{x^2+2}dx$ (This one is a little tricky)

   (b) $\int \frac{2x}{(x+1)(2x-2)}dx$

(c) $\displaystyle\int \frac{x^3 - 2x^2 + 3x}{x^2 - 1} dx$

(d) $\displaystyle\int \frac{1}{x^3 + x} dx$

3. Solve the following differential equations:

   (a) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y,\ x > 0$

   (b) $e^x \frac{dy}{dx} + 2e^x y = 1$

   (c) $\frac{dy}{dt} + 2y = 3,\ y(0) = 1$

   (d) $\theta\frac{dy}{d\theta} + y = \sin\theta,\ \theta > 0,\ y\left(\frac{\pi}{2}\right) = 1$

4. Calculate the following integrals:

   (a) $\displaystyle\int \frac{dy}{y^2 - 2y + 2}$

   (b) $\displaystyle\int \ln\sqrt{x-1}dx$

   (c) $\displaystyle\int \frac{x+1}{x^2(x^2+1)}dx$

   (d) $\displaystyle\int x^3 e^{x^2} dx$

   (e) $\displaystyle\int \sin^2 x dx$

   (f) $\displaystyle\int \frac{e^t dt}{1 + e^t}$

   (g) $\displaystyle\int \frac{\cot v dv}{\ln(\sin v)}$

   (h) $\displaystyle\int e^x \cos(2x) dx$

   (i) $\displaystyle\int \frac{dx}{(x^2-1)^{\frac{3}{2}}}$

5. Determine if the following integrals converge or diverge. Give reasons for your answers. If the integral converges, find it's value if possible.

   (a) $\displaystyle\int_0^1 \ln x dx$

$$= \lim_{t\to 0} \int_t^1 \ln x dx$$

$$u = \ln x \qquad dv = dx$$

$$du = \frac{1}{x}dx \qquad v = x$$

$$= \lim_{t\to 0}\left(x\ln x - \int x \cdot \frac{1}{x}dx\right)$$

$$= \lim_{t\to 0}(x\ln x - x|_t^1$$

$$= \lim_{t\to 0} -1 - t\ln t + t$$

$$= -1 - \lim_{t\to 0} \frac{\ln t}{\frac{1}{t}}$$

$$=^{LH} -1 - \lim_{t\to 0} \frac{\frac{1}{t}}{-\frac{1}{t^2}}$$

$$= -1 + \lim_{t\to 0} \frac{t^2}{t}$$

$$= -1 - \lim_{t\to 0} t$$

$$= 1$$

Therefore the integral converges.

(b) $\int_3^5 \frac{1}{x-4}dx$

$$= \int_3^4 \frac{1}{x-4}dx + \int_4^5 \frac{1}{x-4}dx$$

We will do the first integral first:

$$\int_3^4 \frac{1}{x-4}dx = \lim_{t\to 4^-} \int_3^t \frac{1}{x-4}dx$$

$$= \lim_{t\to 4^-} (\ln\mid x-4\mid|_3^t$$

$$= \lim_{t\to 4^-} \ln\mid t-4\mid - \ln\mid -1\mid$$

$$= \lim_{t\to 4^-} \ln\mid t-4\mid -0$$

$$= -\infty$$

Therefore, since one part of the integral diverges, the entire integral diverges.

(c) $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$

$$= \lim_{t\to 3^-} \int_0^t \frac{dx}{\sqrt{9-x^2}}$$

$$= \lim_{t\to 3^-} \left(\frac{1}{3}\sin^{-1}\left(\frac{x}{3}\right)\right)_0^t$$

$$= \lim_{t\to 3^-} \frac{1}{3}\sin^{-1}\left(\frac{t}{3}\right) - \frac{1}{3}\sin^{-1} 0$$

$$= \frac{1}{3}\frac{\pi}{2} - 0$$

$$= \frac{\pi}{6}$$

so the integral converges.

(d) $\int_0^\infty \frac{2dx}{x^2-2x}$

$$= \int_0^1 \frac{2dx}{x^2-2x} + \int_1^2 \frac{2dx}{x^2-2x} + \int_2^3 \frac{2dx}{x^2-2x} + \int_3^\infty \frac{2dx}{x^2-2x}$$

We will look at the last one:

$$\int_3^\infty \frac{2dx}{x^2-2x} = \lim_{t\to\infty} 2\int_3^t \frac{dx}{x(x-2)}$$

$$= \lim_{t\to\infty} 2\int_3^t \frac{A}{x} + \frac{B}{x-2}dx$$

solving for the coefficients:

$$A(x-2) + Bx = 1$$

If $x = 0 \Rightarrow -2A = 1 \Rightarrow A = -\frac{1}{2}$
If $x = 2 \Rightarrow 2B = 1 \Rightarrow B = \frac{1}{2}$

$$= \lim_{t\to\infty} 2\int_3^t \frac{-\frac{1}{2}}{x} + \frac{\frac{1}{2}}{x-2}dx$$

$$= \lim_{t\to\infty} 2\left(-\frac{1}{2}\ln\mid x\mid + \frac{1}{2}\ln\mid x-2\mid\right)_3^t$$

$$= \lim_{t\to\infty} (-\ln\mid x\mid + \ln\mid x-2\mid)_3^t$$

$$= \lim_{t\to\infty} \left(\ln\left|\frac{x-2}{x}\right|\right)_3^t$$

$$= \lim_{t\to\infty} \ln\left|\frac{t-2}{t}\right| - \ln\frac{1}{3}$$

$$= \ln 1 - \ln\frac{1}{3} = -\ln\frac{1}{3}$$

Let us look at

$$\int_0^1 \frac{2dx}{x^2-2x} = \lim_{t\to 0^+} \int_t^1 \frac{2dx}{x^2-2x}$$

$$= \lim_{t\to 0^+} \int_t^1 \frac{-\frac{1}{2}}{x} + \frac{\frac{1}{2}}{x-2}dx$$

$$= \lim_{t\to 0^+} \left(\ln\left|\frac{x-2}{x}\right|\right)_t^1$$

$$= \lim_{t\to 0^+} \ln\left|\frac{-1}{1}\right| - \ln\left|\frac{t-2}{t}\right|$$

$$= 0 - \lim_{t\to 0^+} \ln\left|\frac{t}{t} - \frac{2}{t}\right|$$

$$= 0 - \infty = -\infty$$

So this integral diverges.

(e) $\int_1^\infty \frac{3x-1}{4x^3-x^2}$

$$= \lim_{t\to\infty} \int_1^t \frac{3x-1}{x^2(2x-1)} dx$$
$$= \lim_{t\to\infty} \int_1^t \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x-1} dx$$

solving for the coefficients:

$$Ax(2x-1) + B(2x-1) + Cx^2 = 3x-1$$

If $x = 0 \Rightarrow -B = -1 \Rightarrow B = 1$
If $x = \frac{1}{2} \Rightarrow \frac{1}{4}C = \frac{3}{2} - 1 = \frac{1}{2} \Rightarrow C = 2$

$$2Ax^2 - Ax + 2Bx - B + Cx^2 = 3x - 1$$
$$(2A + C)x^2 - (2B - A)x - B = 3x - 1$$
$$(2A + 2)x^2 + (A - 2)x - 1 = 3x - 1$$
$$A - 2 = 3 \Rightarrow A = 5$$

$$= \lim_{t\to\infty} \int_1^t \frac{5}{x} + \frac{1}{x^2} + \frac{2}{2x-1} dx$$
$$= \lim_{t\to\infty} \left(5 \ln \mid x \mid - \frac{1}{x} + \ln \mid 2x - 1 \mid\right)_1^t$$
$$= \lim_{t\to\infty} 5 \ln \mid t \mid - \frac{1}{t} + \ln \mid 2x - 1 \mid - 0 + 1 + 0$$
$$= \infty$$

so the integral diverges.

(f) $\int_0^\infty x^2 e^{-x} dx$

$$= \lim_{t\to\infty} \int_0^t x^2 e^{-x} dx$$
$$u = x^2 \qquad dv = e^{-x} dx$$
$$du = 2x dx \qquad v = -e^{-x}$$
$$= \lim_{t\to\infty} -x^2 e^{-x} + 2 \int_0^t x e^{-x} dx$$
$$u = x \qquad dv = e^{-x} dx$$
$$du = dx \qquad v = -e^{-x}$$
$$= \lim_{t\to\infty} -x^2 e^{-x} + 2\left(-x e^{-x} + \int_0^t e^{-x} dx\right)$$
$$= \lim_{t\to\infty} \left(-x^2 e^{-x} - 2x e^{-x} - 2e^{-x}\right)_0^t$$
$$= \lim_{t\to\infty} \left(-t^2 e^{-t} - 2t e^{-t} - 2e^{-t}\right) - (-2) = 0 + 2 = 0$$

so the integral converges.

(g) $\int_1^\infty \frac{e^{-t}}{\sqrt{t}}dt$

$$0 < \frac{e^{-t}}{\sqrt{t}} \le e^{-t} \text{ for } t \ge 1$$

$$\int_1^\infty e^{-t} = \lim_{s\to\infty} -e^{-t} \,|_1^s$$

$$= \lim_{s\to\infty} -e^{-s} + \frac{1}{e}$$

$$= \frac{1}{e}$$

Since the larger integral converges, $\int_1^\infty \frac{e^{-t}}{\sqrt{t}}dt$ also converges, although we can't find it's value.