

Here are some good review questions. The actual exam might be of a different format, but these will help you understand the concepts covered on the exam. Try to do as many of these as you can without looking in your notes or book for guidance.

1. State the fundamental theorem of calculus (both parts, include the appropriate hypotheses).
2. Explain what the fundamental theorem of calculus means (i.e. why is it important).
3. State the definition of integrable.
4. Explain the difference, if any, between an anti-derivate and an indefinite integral.
5. State the Mean Value Theorem for Definite Integrals.
6. Know the formulas necessary to evaluate $\int \sin^2 x dx$ and $\int \cos^2 x dx$.
7. Find the average value of $f(x)$ over the given interval:
 - (a) $f(x) = -x^2 + 10x + 11$ on the interval $[0, 10]$
8. Let $f(t) = \frac{1}{2} \sin^2 t$. Then $F'(t) = \sin t \cos t$. Use the Fundamental Theorem of Calculus to find $\int_{\frac{\pi}{2}}^{\pi} \sin t \cos t dt$.
9. If the average value of $f(x)$ over $[-3, 5]$ is 4, what is $\int_{-3}^5 f(x) dx$?
10. Suppose that $v(t) = 3t^2 + 6t + 10$ gives the velocity in miles per hour after t hours.
 - (a) Use a Riemann sum to estimate the total distance traveled between $t = 3$ and $t = 5$. Use 4 subintervals and the midpoint of each subinterval.
 - (b) Find the exact distance traveled between $t = 3$ and $t = 5$.
11. Compute the following indefinite integrals:
 - (a) $\int \frac{\csc \theta}{\csc \theta - \sin \theta} d\theta$
 - (b) $\int \frac{e^x}{1+e^{2x}} dx$
 - (c) $\int x^{-2} + x^3 + 2x + 5 dx$
 - (d) $\int e^x + x^e + e dx$
 - (e) $\int \frac{1}{x} dx$
 - (f) $\int 3 \sin x - 5 \cos x dx$
 - (g) $\int x(x^2 - 3)^{49} dx$
 - (h) $\int \frac{x-2}{x^2-4x} dx$
 - (i) $\int x \sin(x^2) dx$
12. Compute the following derivatives:
 - (a) $\frac{d}{dx} \int_1^x (t^2 - 1)^{19} dt$
 - (b) $\frac{d}{dx} \int_0^x \sin \theta^2 d\theta$
 - (c) $\frac{d}{dx} \int_x^{\pi} \frac{1}{1+t^4} dt$
 - (d) $\frac{d}{dx} \int_1^{\sqrt{x}} \frac{r^2}{r^2+1} dr$
 - (e) $\frac{d}{dx} \int_{-5}^{\sin x} u \cos u^3 du$

(f) $\frac{d}{dx} \int_2^{129} \sin x dx$

13. Compute the following definite integrals:

(a) $\int_{-4}^6 |x + 2| dx$

(b) $\int_0^{\frac{\pi}{2}} e^{\sin x} \cos x dx$

(c) $\int_0^{10} -x^2 + 10x + 11 dx$

(d) $\int_1^2 \sqrt{x} dx$

(e) $\int_2^6 3t^2 + 4t dt$

(f) $\int_0^3 x(x^2 - 3)^{49} dx$

(g) $\int_1^4 \frac{x-2}{x^2-4x} dx$

(h) $\int_{\frac{\pi}{2}}^{\pi} x \sin(x^2) dx$

14. Suppose that f and g are continuous functions and that $\int_0^2 f(x) dx = \sqrt{2}$, $\int_0^5 f(x) dx = \sqrt{5}$ and $\int_0^2 g(x) dx = 1$. Find the following:

(a) $\int_0^2 (7f(x) - 11g(x)) dx$

(b) $\int_2^5 f(x) dx$

15. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x -axis and the line $y = x - 2$.16. Find the area of the region bounded by the curves $y = x^3$ and $x = y^3$.17. Calculate the volume of the solid of revolution obtained by rotating the region below about the x -axis

(a) the region bounded by the curves $y = \sqrt{x}$ and $y = \frac{1}{2}x$.

(b) the region bounded by the curves $y = x^2 + 1$ and $y = 2 - 3x^2$

(c) the region bounded by the curve $y = \sqrt{x-1}$, the x -axis, the y -axis, and the line $y = 2$

18. Find the volume of the solids obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the following lines:

(a) the x -axis

(b) the y -axis

(c) the line $y = 2$

(d) the line $x = 2$

19. Find the volume of the solids obtained by rotating the region bounded by the curves $y = \sin x$, $y = 0$, $x = 0$, and $x = \pi$ about the y -axis.20. Compute $\frac{d}{dx} \int_{e^x}^e \ln t dt$. Simplify.

21. A force of 30 N is required to maintain a spring stretched from its natural length of 12 cm to a length of 15 cm. How much work is done in stretching the spring from 12 cm to 24 cm?

22. Calculate the average value of the function $f(x) = x^2 - 2x$ on the interval $[0, 3]$. Find c such that $f_{avg} = f(c)$. What theorem guarantees that I can find such a c ?23. Know the formulas necessary to evaluate $\int \sin^2 x dx$ and $\int \cos^2 x dx$.

24. Know the integrals of the following functions:

(a) $\int \frac{1}{u} du =$

(b) $\int u^n du$

(c) $\int \sin u du$

(d) $\int \cos u du$

(e) $\int \sin(nu) du$ for n a number

(f) $\int \cos(nu) du$ for n a number

(g) $\int \sec^2 u du$

(h) $\int \sec u \tan u du$

(i) $\int \frac{1}{a^2+u^2} du$

(j) $\int \frac{1}{\sqrt{a^2-x^2}} du$

(k) $\int \csc^2 u du$

(l) $\int \csc u \cot u du$

(m) $\int \tan u du$

(n) $\int e^u du$

(o) $\int a^u du$

(p) $\int \frac{1}{u\sqrt{u^2-a^2}} du$

25. Calculate the following integrals:

(a) $\int \csc u du$

(b) $\int x(x^2 - 3)^{49} dx$

(c) $\int \frac{x-2}{x^2-4x} dx$

(d) $\int x \sin(x^2) dx$

26. Find the length of the curve $y = \ln(\sec x)$ from $x = 0$ to $x = \frac{\pi}{4}$

27. State the definition of $\ln x$ which uses calculus.

28. Solve the following differential equations:

(a) $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y, x > 0$

(b) $e^x \frac{dy}{dx} + 2e^x y = 1$

(c) $\frac{dy}{dt} + 2y = 3, y(0) = 1$

(d) $\theta \frac{dy}{d\theta} + y = \sin \theta, \theta > 0, y\left(\frac{\pi}{2}\right) = 1$