Math 143
Fall, 2005
Final Exam

Part I - Definitions and Examples

1. (3 points) State the definition of an integrable function, i.e, f is integrable on $[a,b]$ if:

2. (3 points) State both parts of the Fundamental Theorem of Calculus.

3. (3 points) State the Monotonic Sequence Theorem.

4. (3 points) State the definition of $\ln x$ using calculus.

   Part II - Calculations

5. (8 points) Use a Riemann sum with $n = 5$ and the left endpoint of each subinterval chose to estimate the area under the curve $y = x^2$ on the interval $[0, 10]$. Is this an overestimate or an underestimate? Why?

6. Find the following integrals:

   (a) (6 points) $\int 4x \sec^2 2x \; dx$

   (b) (7 points) $\int \frac{e^t \; dt}{e^{2t} + 3e^t + 2}$

   (c) (6 points) $\int \frac{2 \; dx}{x^3\sqrt{x^2-1}}$

   (d) (6 points) $\int_0^3 \frac{1}{x-1} \; dx$

   (e) (7 points) $\int_0^\infty x^2 e^{-x} \; dx$

7. (5 points) Set up, but **do not integrate** the integral to find the volume of the solid generated by revolving the region in the first quadrant bounded by the $x-$axis and the curve $y = 3x\sqrt{1-x}$ about the $y-$axis.

8. (6 points) Find the length of the curve $y = \int_0^x \sqrt{\cos 2t} \; dt$ for $0 \le x \le \frac{\pi}{4}$

9. (6 points) Find the first three non-zero terms of the Taylor polynomial for $f(x) = \frac{x}{1-x}$ near $x = 0$.

10. Determine if the following series converge or diverge. Provide reasons for your answers.

   (a) (6 points) $\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$

   (b) (6 points) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

11. (7 points) Find the interval of convergence for the series $\sum_{n=1}^{\infty} \frac{(x-1)^{2n-2}}{(2n-1)!}$

12. Determine if the following series converge absolutely, converge conditionally or diverge. Provide reasons for your answers.

   (a) (6 points) $\sum_{n=1}^{\infty} \frac{(-1)^n 3n^2}{n^3+1}$

   (b) (6 points) $\sum_{n=1}^{\infty} \frac{(-1)^n (n^2+1)}{2n^2+n-1}$