

**Part I - Definitions and Examples**

1. (5 points) State the definition of an integrable function.
2. (5 points) State the Fundamental Theorem of Calculus, part II. Include all necessary hypotheses.
3. (5 points) Explain why this part of the Fundamental Theorem of Calculus is important. (i.e., what makes it fundamental?)
4. (4 points) State the definition of  $\ln x$  which uses calculus.

**Part II - Calculations**

5. (6 points) Find  $\frac{d}{dx} \int_{\sin x}^0 \frac{1}{1+t^2} dt$
6. (8 points) Find  $\int x^e + \pi^e + e^\pi + e^{\pi x} dx$
7. (6 points) Find  $\int \sin x + \frac{1}{1+x^2} + \sec^2 x dx$
8. (6 points) Find  $\int_2^6 |x - 3| dx$
9. (6 points)  $\int 10^{2\theta} d\theta$
10. (6 points)  $\int \sin^3 x \cos x dx$
11. (6 points)  $\int x \tan(x^2) dx$
12. (8 points) Find the area of the region bounded by  $y = x^3$  and  $x = y^3$
13. Solve the following differential equations:
  - (a) (10 points)  $e^x \frac{dy}{dx} + 2e^x y = 1$        $y(0) = 5$
  - (b) (8 points)  $\frac{dy}{dx} = \frac{\sin y}{\cos x}$
14. (8 points) Use a Riemann sum with  $n = 4$  subintervals, and the right endpoint of each subinterval, to approximate the area under the curve  $y = x^2 + 1$  on the interval  $[0, 2]$ . Is this an overestimate or an underestimate? Why? (A picture would work for this part.)
15. (5 points) Set up but do not integrate the integral to compute the volume of the solid generated by rotating the region bounded by the curves  $y = x$  and  $y = x^3$  about the line  $x = 3$ .
16. (5 points) Set up but do not integrate the integral to compute the volume of the solid generated rotating the region bounded by the curves  $y = 8x - x^4$ ,  $x = 0$ , and  $y = 7$  about the  $y$ -axis.
17. (10 points) Find the length of the curve given by  $x = \frac{t^2}{2}$ ,  $y = \frac{(2t+1)^{\frac{3}{2}}}{3}$  from  $t = 0$  to  $t = 3$ .
18. (10 points) Solve the differential equation  $\sqrt{x} \frac{dy}{dx} = e^{y+\sqrt{x}}$ .

**Part III - Applications**

19. (10 points) A force of 30 N is required to maintain a spring stretched from its natural length of 12 cm to a length of 15 cm. How much work is done in stretching the spring from 12 cm to 24 cm?