
Here are some things to look at as you prepare for the final exam. Remember to look at review sheets for other exams, previous exams, and quizzes.

1. Know the statement of the following theorems and definitions:
   (a) The definition of the definite integral of f on $[a, b]$
   (b) The Mean Value Theorem for Definite Integrals
   (c) The Fundamental Theorem of Calculus, Parts I and II
   (d) The definition of $\ln x$ using calculus
   (e) The Sandwich Theorem for Sequences
   (f) Monotonic Sequence Theorem
   (g) $n-$th term test
   (h) The Integral Test
   (i) The Comparison Test for Series
   (j) The Ratio Test
   (k) The Root Test
   (l) Leibniz's theorem / Alternating Series Test
   (m) The Absolute Convergence Test
   (n) The Convergence Theorem for Power Series

2. Be able to do the following integrals:
   (a) $\displaystyle\int \frac{dx}{9-x^2}$
   (b) $\displaystyle\int \frac{dx}{\sqrt{9-x^2}}$
   (c) $\displaystyle\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$
   (d) $\displaystyle\int \frac{dx}{\sqrt{-2x-x^2}}$
   (e) $\displaystyle\int \frac{2-\cos x+\sin x}{\sin^2 x}dx$
   (f) $\displaystyle\int \theta\cos(2\theta+1)d\theta$
   (g) $\displaystyle\int \frac{x^3}{x^2-2x+1}dx$
   (h) $\displaystyle\int \frac{dx}{\sqrt{1+\sqrt{x}}}$

(i) $\int \frac{2\sin\sqrt{x}dx}{\sqrt{x}\sec\sqrt{x}}$

(j) $\int \frac{\sin 2x dx}{(1+\cos 2x)^2}$

(k) $\int \frac{dy}{y^2-2y+2}$

(l) $\int \ln\sqrt{x-1}dx$

(m) $\int \frac{x+1}{x^2(x^2+1)}$

(n) $\int x^3 e^{x^2} dx$

(o) $\int \sin^2 x dx$

(p) $\int \frac{\cos(\sin^- 1x)}{\sqrt{1-x^2}}$

(q) $\int \frac{e^t dt}{1+e^t}$

(r) $\int \frac{\cot v dv}{\ln(\sin v)}$

(s) $\int (27)^{3\theta+1}\, d\theta$

(t) $\int e^x \cos(2x)dx$

(u) $\frac{dx}{x^2-3x+2}$

(v) $\int \frac{dx}{(x^2-1)^{\frac{3}{2}}}$

3. Determine if the following integrals converge or diverge. Give reasons for your answers. If the integral converges, find it's value if possible.

(a) $\int_0^1 \ln x dx$

(b) $\int_3^5 \frac{1}{x-4}dx$

(c) $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$

(d) $\int_0^\infty \frac{2dx}{x^2-2x}$

(e) $\int_1^\infty \frac{3x-1}{4x^3-x^2}$

(f) $\int_0^\infty x^2 e^{-x} dx$

(g) $\int_1^\infty \frac{e^{-t}}{\sqrt{t}} dt$

4. Calculate the following limits:

   (a) $\lim_{x \to 0} \frac{x \sin x}{1 - \cos x}$

   (b) $\lim_{x \to \infty} x^{\frac{1}{1-x}}$

   (c) $\lim_{t \to 0} \frac{\tan 3t}{\tan 5t}$

5. Solve the following differential equations

   (a) $\frac{dy}{dx} = -\frac{y \ln y}{1 + x^2}$ where $y(0) = e^2$

   (b) $\frac{dy}{dx} + \left(\frac{2}{x+1}\right) y = \frac{x}{x+1}$

   (c) $x\frac{dy}{dx} + 2y = x^2 + 1$

   (d) $xy' + y = x \cos x$

6. Find the volume of the solid generated by revolving the region bounded by the $x-$axis, the curve$y = 3x^4$ and the lines $x = 1$ and $x = -1$ about

   (a) the $x-$axis

   (b) the $y-$axis

   (c) the line $x = 1$

   (d) the line $y = 3$

7. Find the area between the curves $x = \frac{y^2}{4}$ and $y = x$

8. Find the lengths of the following curves on the given intervals;

   (a) $x = y^{\frac{2}{3}}, 1 \leq y \leq 8$

   (b) $x = 5 \cos t - \cos 5t, y = 5 \sin t - \sin 5t, 0 \leq t \leq \frac{\pi}{2}$

9. A force of 200 N will stretch a garage door spring 0.8 m beyond it unstressed length. How far will a 300 N force stretch the spring? How much work does it take to stretch the spring this far from its unstressed length?

10. Find the limits of the following sequences:

    (a) $a_n = \left(1 + \frac{(-1)^n}{n}\right)$

    (b) $a_n = \sin \frac{n\pi}{2}$

    (c) $a_n = \frac{n + \ln n}{n}$

    (d) $a_n = \frac{\ln(2n^3)}{n}$

(e) $a_n = \dfrac{(n+1)!}{n!}$

(f) $a_n = n\left(2^{\frac{1}{n}} - 1\right)$

11. Determine if the following series converge or diverge. Provide reasons for your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-3)(2n-1)}$

(b) $\displaystyle\sum_{n=0}^{\infty} e^{-n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{-5}{n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n^3}$

12. Determine if the following series converge absolutely, converge conditionally or diverge. Provide reasons for your answers.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n(\ln n)^2}$

(b) $\displaystyle\sum_{n=2}^{\infty} (-1)^{n+1} \frac{\ln n}{n^3}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n} \frac{n+1}{n!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n^2+1)}{2n^2+n-1}$

13. Determine the center and radius of convergence for the following power series. Specify where they converge absolutely and where they converge conditionally.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^{2n-2}}{(2n-1)!}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(3x-1)^n}{n^2}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^n}$

14. Find Maclaurin series for the following functions: $\dfrac{1}{1-2x}$, $\cos\left(x^{\frac{5}{2}}\right)$, and $e^{\frac{\pi x}{2}}$

15. Find the first four nonzero terms f the Taylor series generated by each of the following functions at x=2: $f(x) = \sqrt{3+x^2}$, $g(x) = \dfrac{1}{x+1}$, and $h(x) = \dfrac{1}{x}$

16. Remember to also study polar equations, including area integrals and arc length integrals.