Math 142
Spring, 2004
Review of Derivatives

Name: **Solutions**

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1. Calculate the derivatives of the following functions. Wherever necessary, assume that y is a function of x.

   (a) $y = \sqrt{x^2 + e^x}$

$$y' = \frac{1}{2}\left(x^2 + e^x\right)^{-\frac{1}{2}}(2x + e^x)$$

   (b) $y = \ln x^{500}$

$$y = 500 \ln x$$
$$y' = 500 \cdot \frac{1}{x} = \frac{500}{x}$$

   (c) $\cos(xy) = \sin(xy)$

$$-\sin(xy)\left(y + x\frac{dy}{dx}\right) = \cos(xy)\left(y + x\frac{dy}{dx}\right)$$
$$-y\sin(xy) - x\sin(xy)\frac{dy}{dx} = y\cos(xy) + x\cos(xy)\frac{dy}{dx}$$
$$-y\sin(xy) - y\cos(xy) = x\cos(xy)\frac{dy}{dx} + x\sin(xy)\frac{dy}{dx}$$
$$-y\sin(xy) - y\cos(xy) = (x\cos(xy) + x\sin(xy))\frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{-y\sin(xy) - y\cos(xy)}{x\cos(xy) + x\sin(xy)}$$

   (d) $\frac{y^2 + 5\tan x - \frac{x}{y}}{x^4} = x$

$$y^2 + 5\tan x - \frac{x}{y} = x^5$$
$$2y\frac{dy}{dx} + 5\sec^2 x - \frac{y - x\frac{dy}{dx}}{y^2} = 5x^4$$
$$2y\frac{dy}{dx} + 5\sec^2 x - \frac{1}{y} + \frac{x}{y^2}\frac{dy}{dx} = 5x^4$$
$$\left(2y + \frac{x}{y^2}\right)\frac{dy}{dx} = 5x^4 - 5\sec^2 x + \frac{1}{y}$$
$$\frac{dy}{dx} = \frac{5x^4 - 5\sec^2 x + \frac{1}{y}}{2y + \frac{x}{y^2}}$$

(e) $24x^3 - 3x^{-7} = y$

$$y' = 72x^2 + 21x^{-8}$$

(f) $f(x) = (e^x + \arcsin x)(\frac{3}{x^2})$

$$f'(x) = \left(e^x + \frac{1}{\sqrt{1-x^2}}\right)\left(\frac{3}{x^2}\right) + \left(e^x + \sin^{-1} x\right)\left(\frac{-6}{x^3}\right)$$

(g) $y = x^3 - 3x^2 - 1$

$$y' = 3x^2 - 6x$$

(h) $f(x) = 7x - 3$

$$f'(x) = 7$$

(i) $f(x) = e^\pi$

$$f'(x) = 0$$

(j) $f(x) = \frac{1}{\sqrt{x}}$

$$f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$$

(k) $f(x) = 5\sqrt[3]{x} + 6x^2 - e$

$$f'(x) = \frac{5}{3}x^{-\frac{2}{3}} + 12x$$

(l) $f(x) = \frac{1}{x^2} + \frac{1}{x} + 1$

$$f'(x) = -\frac{2}{x^3} - \frac{1}{x^2}$$

(m) $f(x) = \sin 3$

$$f'(x) = 0$$

(n) $f(x) = x^2 \sin 2$

$$f'(x) = (2\sin 2)x$$

(o) $f(x) = x^e + x^\pi$

$$f'(x) = ex^{e-1} + \pi x^{\pi-1}$$

2. Compute the following derivatives:

   (a) $\frac{d}{dx}(\frac{x^3}{2})$

$$= \frac{3}{2}x^2$$

   (b) $\frac{d}{dx}\left(\frac{3x+2}{2x-11}\right)$

$$= \frac{3(2x-11) - 2(3x+2)}{(2x-11)^2} = \frac{6x - 33 - 6x - 4}{(2x-11)^2} = \frac{-37}{(2x-11)^2}$$

   (c) $\frac{d}{dx}(\sqrt{\sin^2 x})$

$$= \frac{d}{dx}\sin x = \cos x$$

   (d) $\frac{d}{dx}\left((\sin^2(2x) + 1)^e\right)$

$$= e(\sin^2(2x) + 1)^{e-1}(2\sin(2x) \cdot \cos(2x) \cdot 2)$$
$$= 4e\sin(2x)\cos(2x)(\sin^2(2x) + 1)^{e-1}$$

   (e) $\frac{d}{dx}(\tan(\cos(\sin x)))$

$$= \sec^2(\cos(\sin x))(-\sin(\sin x))(\cos x)$$

   (f) $\frac{d}{dx}((\tan x)(\cos x)(\sin x))$

$$= sec^2 x\cos x \sin x - \tan x \sin^2 x + \tan x \cos^2 x$$

   (g) $\frac{d}{dx}\left((4x+3)^4(x+1)^{-1}\right)$

$$= 4(4x+3)^3 \cdot 4 \cdot (x+1)^{-1} - (x+1)^{-2}(4x+3)^4$$

   (h) $\frac{d}{dx}\left(\sin\left(\frac{3\pi x}{x}\right) + \cos\left(\frac{3\pi x}{2}\right)\right)$

$$= -\sin\left(\frac{3\pi x}{2}\right) \cdot \frac{3\pi}{2}$$

   (i) $\frac{d}{dx}(x\tan x)$

$$= \tan x + x\sec^2 x$$

   (j) $\frac{d}{dx}\left((x^2+1)\sec x\right)$

$$= 2x\sec x + (x^2+1)\sec x \tan x$$

(k) $\frac{d}{dx}\left(x(x^2+1)\tan x \sec x\right)$

$$\begin{aligned} &= \frac{d}{dx}\left((x^3+x)\tan x \sec x\right) \\ &= (3x^2+1)\tan x \sec x + (x^3+x)\sec^3 x + (x^3+x)\sec x \tan^2 x \end{aligned}$$

(l) $\frac{d}{dx}\left(x\sin^{-1} x + \sqrt{-x^2}\right)$

$$\begin{aligned} &= \sin^{-1} x + \frac{x}{\sqrt{1-x^2}} + \frac{1}{2}(-x^2)^{-\frac{1}{2}}(-2x) \\ &= \sin^{-1} x + \frac{x}{\sqrt{1-x^2}} + -\frac{x}{\sqrt{-x^2}} \end{aligned}$$

(m) $\frac{d}{dt}\left(\cot^{-1}\sqrt{t-1}\right)$

$$\begin{aligned} &= -\frac{1}{1+\sqrt{t-1}^2}\left(\frac{1}{2}(t-1)^{-\frac{1}{2}}\right) = \frac{1}{t}\frac{1}{2}\frac{1}{\sqrt{t-1}} \\ &= \frac{1}{2t\sqrt{t-1}} \end{aligned}$$

(n) $\frac{d}{ds}\left(\tan^{-1}\sqrt{x^2-1} + \sin^{-1} x\right)$

$$\begin{aligned} &= \frac{1}{1+\sqrt{x^2-1}^2}\frac{1}{2}(x^2-1)^{-\frac{1}{2}} + \frac{1}{\sqrt{1-x^2}} \\ &= \frac{1}{1+x^2-1}\frac{1}{2\sqrt{x^2-1}} + \frac{1}{\sqrt{1-x^2}} \\ &= \frac{1}{2x^2\sqrt{x^2-1}} + \frac{1}{\sqrt{1-x^2}} \end{aligned}$$

(o) $\frac{d}{dx}\left(3^{\sin x}\right)$

$$= 3^{\sin x}\ln 3 \cos x$$

(p) $\frac{d}{dx}\left(\ln(x^2+1)\right)$

$$= \frac{2x}{x^2+1}$$

(q) $\frac{d}{dx}\left(e^{\sin x}\right)$

$$= e^{\sin x}\cos x$$

(r) $\frac{d}{dx}\left(x^{\sin x}\right)$

$$\begin{aligned} y &= x^{\sin x} \\ \ln y &= \sin x \ln x \end{aligned}$$

$$\frac{1}{y}y' = \cos x \ln x + \frac{\sin x}{x}$$
$$y' = x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)$$

(s) $\frac{d}{dx}\left(\sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}\right)$

$$y = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}$$
$$\ln y = \ln \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}$$
$$\ln y = \frac{1}{5}\ln \frac{(x-3)^4(x^2+1)}{(2x+5)^3}$$
$$\ln y = \frac{1}{5}\left(\ln(x-3)^4(x^2+1) - \ln(2x+5)^3\right)$$
$$\ln y = \frac{1}{5}\left(\ln(x-3)^4 + \ln(x^2+1) - 3\ln(2x+5)\right)$$
$$\ln y = \frac{4}{5}\ln(x-3) + \frac{1}{5}\ln(x^2+1) - \frac{3}{5}\ln(2x+5)$$
$$\frac{1}{y}y' = \frac{4}{5}\frac{1}{x-3} + \frac{1}{5}\frac{2x}{x^2+1} - \frac{3}{5}\frac{2}{2x+5}$$
$$y' = y\left(\frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x+5)}\right)$$
$$y' = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}\left(\frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x+5)}\right)$$

(t) $\frac{d}{dx}\left(x\sin x + e^x + \pi^x + x^\pi\right)$

$$= \sin x + x\cos x + e^x + \pi^x \ln \pi + \pi x^{\pi-1}$$

(u) $\frac{d}{dx}\left(\ln 2 \cdot \log_2 x\right)$

$$= \frac{d}{dx}\left(\ln 2 \cdot \frac{\ln x}{\ln 2}\right)$$
$$= \frac{d}{dx}\left(\ln x\right)$$
$$= \frac{1}{x}$$

(v) $\frac{d}{dx}\left(\ln 10^x\right)$

$$= \frac{d}{dx}\left(x \ln 10\right)$$
$$= \ln 10$$

(w) $\dfrac{d}{dt}\left(\dfrac{t^2+3t}{t}\right)$

$$= \frac{(2t+3)t-(t^2+3t)}{t^2}$$
$$= \frac{2t^2+3t-t^2-3t}{t^2}$$
$$= \frac{t^2}{t^2}$$
$$= 1$$

(x) $\frac{d}{dx}\left(x\ln x\right)$

$$= \ln x + 1$$

(y) $\dfrac{d}{dx}\left(\dfrac{e^x}{e^{-x}+1}\right)$

$$= \frac{e^x(e^{-x}+1)-(-e^{-x})(e^x)}{(e^{-x}+1)^2}$$
$$= \frac{1+e^x+1}{(e^{-x}+1)^2}$$
$$= \frac{2+e^x}{(e^{-x}+1)^2}$$