Math 143, Fall, 2006 **Review of Integrals** Name:

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1. Find the most general antiderivative for the following functions:
   (a) $f(x) = 5x^{\frac{1}{4}} - 7x^{\frac{3}{4}}$
   (b) $f(t) = 7\cos t - 5\sin t$
   (c) $f(\theta) = e^{\theta} + \sec\theta\tan\theta$
   (d) $f(x) = \dfrac{x^2 + x + 1}{x}$
2. Find $f(x)$:
   (a) $f'(x) = \dfrac{4}{\sqrt{1-x^2}}$ when $f(\frac{1}{2}) = 1$
   (b) $f''(x) = x^2 + 3\cos x$ when $f(1) = 1$ and $f'(1) = 5$
3. Find all possible functions with the given derivatives:
   (a) $f'(x) = \dfrac{1}{x^2} + \dfrac{1}{x} + 1 + x + x^2$
   (b) $f'(x) = \sec x\tan x$
   (c) $f'(x) = 5^x + xe^{-x^2}$
   (d) $f'(x) = \sin x$
   (e) $f'(x) = \sqrt{2x+3} + \dfrac{1}{\sqrt{2x+3}}$
4. Evaluate the following integrals. Use differentiation to check your answers:
   (a) $\displaystyle\int xe^{-x^2}dx$
   (b) $\displaystyle\int \cos x dx$
   (c) $\displaystyle\int \frac{3}{(2-x)^2}dx$
   (d) $\displaystyle\int \tan x dx$
   (e) $\displaystyle\int \frac{1}{\sqrt{1-3x}}dx$
   (f) $\displaystyle\int \frac{1}{\sqrt{1-9x^2}}dx$
   (g) $\displaystyle\int (e^{-x} + 4^x)dx$
   (h) $\displaystyle\int (\cos^2 x + \sin^2 x)dx$
   (i) $\displaystyle\int \pi dx$
   (j) $\displaystyle\int \frac{4}{x + x\ln^2 x}dx$
   (k) $\displaystyle\int \frac{x^2+4}{x}dx$

5. Find the average value of $f(x)$ over the given interval where $f(x) = -x^2 + 10x + 11$ on the interval $[0, 10]$

6. Let $f(t) = \frac{1}{2}\sin^2 t$. Then $F'(t) = \sin t \cos t$. Use the Fundamental Theorem of Calculus to find $\int_{\frac{\pi}{2}}^{\pi} \sin t \cos t \ dt$.

7. If the average value of $f(x)$ over $[-3, 5]$ is 4, what is $\int_{-3}^{5} f(x)dx$?

8. Compute the following indefinite integrals:

   (a) $\int \frac{\csc\theta}{\csc\theta - \sin\theta} d\theta$

   (b) $\int \frac{e^x}{1 + e^{2x}} dx$

   (c) $\int x^{-2} + x^3 + 2x + 5 \ dx$

   (d) $\int e^x + x^e + e dx$

   (e) $\int \frac{1}{x} dx$

   (f) $\int 3\sin x - 5\cos x \ dx$

   (g) $\int x(x^2 - 3)^{49} \ dx$

   (h) $\int \frac{x - 2}{x^2 - 4x} \ dx$

   (i) $\int x \sin(x^2) \ dx$

9. Compute the following definite integrals:

   (a) $\int_{-4}^{6} | \ x + 2 \ | \ dx$

   (b) $\int_{0}^{\frac{\pi}{2}} e^{\sin x} \cos x \ dx$

   (c) $\int_{0}^{10} -x^2 + 10x + 11 \ dx$

   (d) $\int_{1}^{2} \sqrt{x} \ dx$

   (e) $\int_{2}^{6} 3t^2 + 4t \ dt$

   (f) $\int_{0}^{3} x(x^2 - 3)^{49} \ dx$

   (g) $\int_{1}^{4} \frac{x - 2}{x^2 - 4x} \ dx$

   (h) $\int_{\frac{\pi}{2}}^{\pi} x \sin(x^2) \ dx$

10. Suppose that f and g are continuous functions and that $\int_0^2 f(x)dx = \sqrt{2}$, $\int_0^5 f(x)dx = \sqrt{5}$ and $\int_0^2 g(x)dx = 1$. Find the following:

    (a) $\int_0^2 (7f(x) - 11g(x))dx$

    (b) $\int_2^5 f(x)dx$

11. Compute the following derivatives:

    (a) $\frac{d}{dx}\int_1^x (t^2-1)^{19}dt$

    (b) $\frac{d}{dx}\int_0^x \sin\theta^2 d\theta$

    (c) $\frac{d}{dx}\int_x^\pi \frac{1}{1+t^4}dt$

    (d) $\frac{d}{dx}\int_1^{\sqrt{x}} \frac{r^2}{r^2+1}dr$

    (e) $\frac{d}{dx}\int_{-5}^{\sin x} u\cos u^3 du$

    (f) $\frac{d}{dx}\int_2^{129} \sin x dx$

12. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the $x-$axis and the line $y = x - 2$.

13. Find the area of the region bounded by the curves $y = x^3$ and $x = y^3$.