Math 142
Fall 2007 Name:
Worksheet 9 Group members:

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1. If $f(x) = ax^3 + bx^2 + cx + d$, find values for a, b, c and d so that $f(x)$ has a local maximum at $x = -1$, $f(-1) = 2$, $f(x)$ has a local minimum at $x = 1$ and $f(1) = -1$.
2. Find the critical points, relative maxima and minima for the following:
   (a) $f(x) = x^4 - 4x^3$
   (b) $f(x) = \sqrt{\dfrac{1+x^2}{2+x^2}}$
3. Sand is flowing from a pipe at the rate of s cubic meters per second, and falling in a conical pile. The diameter of the base of this conical pile is always 3 times the altitude. At what rate is the altitude of the pile increasing when the altitude is h meters?
4. (a) An arc light is 5 meters above a sidewalk. A man 2 meters tall walks away from the point under the light at a rate of 2 meters per second. How fast is his shadow lengthening when he is 7 meters away from the point under the light?
   (b) A light is on the ground 40 m away from a building. A man 2 m tall walks from the light towards the building at 2 meters per second. How rapidly is his shadow on the building growing shorter when he is 20 m from the building?
5. Mr. A is walking at 2 mph due south toward point c. Ms. B is walking 3 mph due east towards point c. At a certain instant, Mr. A is 3 miles from c and Ms. B is 4 miles from c.
   (a) At that instant, what is the rate that the distance between Mr. A and Ms. B decreasing?
   (b) At that instant, what is the rate that the angle $\angle ABC$ is changing?
6. Find the maximum slope of the graph $f(x) = 24x^2 - x^4$ for $x \geq 0$.
7. Find constants a and b so that $f(x) = a + bx - x^2$ has a local maximum at $(-1, 2)$.
8. Find the dimensions of the rectangle with area 100 cm whose perimeter is minimized.
9. A cylindrical bottle is to be designed so it holds 100 cubic centimeters of perfume. Find the dimensions of the bottle so the amount of material used in the sides and the bottom is as small as possible. (Hint: A cylinder of height h and radius r has volume $\pi r^2 h$, lateral surface area $2\pi rh$, and surface area of top or bottom πr^2.)
10. Suppose the bottle from the previous problem is to be a rectangular box with square base in shape. Find the dimensions of the bottle which minimizes the amount of material used in the sides and bottom. Which shape uses less material?
11. The demand equation for a certain product is $p = 6 - \frac{1}{2}x$ where x is the number of units sold and p is the price. Find the level of production which results in maximum revenue. (Hint: revenue is price times quantity sold.)

12. The math department needs a substitute professor to give one 12,000 word lecture on calculus. The don't care how long it takes the professor to deliver the lecture, but they do care how much it costs. The different professors speak at different rates of speed, and professors who speak at the rate of v words per minute charge $16 + 10^{-6}v^3$ dollars per hour for their services. How should v be chosen so as to minimize the fee paid for the lecture?

13. Find the dimensions of a closed rectangular box with square base and volume 800 cubic centimeters which is constructed from the least amount of material.

14. A rancher will make 2 corrals from 215 meters of fence. One corral is a square. the other corral is a rectangle with length 1.5 times its width. What are the dimensions of each corral resulting in the greatest combined area?