

1. Determine whether the following statements are true or false. If they are false, find a counterexample:
  - (a) If  $f(-1) = -1$  and  $f(1) = 1$  then  $f(0) = 0$ .
  - (b) If  $f(-1) = -1$  and  $f(1) = 1$ , then there is a point  $c$  so that  $-1 < c < 1$  and  $f(c) = 0$ .
  - (c) If  $f(-1) = -1$ ,  $f(1) = 1$  and  $f(x)$  is continuous, then there is a point  $c$ ,  $-1 < c < 1$  such that  $f(c) = 0$ .
  - (d) If  $f(0) = 0$ ,  $f(1) = 10$  and  $f(x)$  is continuous, then on the interval  $[0, 10]$ ,  $f(x)$  must have a maximum or a minimum value.
2. Let  $f(x) = 3x^2$ .
  - (a) Graph  $f(x)$ .
  - (b) Calculate  $f'(x)$ .
  - (c) Graph  $f'(x)$  on the same axes as  $f(x)$ . What do you notice about the relationship between the graphs?
3. Let  $f(x) = x^3$ .
  - (a) Graph  $f(x)$ .
  - (b) Calculate  $f'(x)$ .
  - (c) Graph  $f'(x)$  on the same axes as  $f(x)$ . What do you notice about the relationship between the graphs?
4. Let  $f(x) = x^2$ .
  - (a) Find the equation of the tangent line to the graph of  $f(x)$  at  $(1, 1)$ .
  - (b) The normal line is the line perpendicular to the tangent line. Find the normal line to the graph of  $f(x)$  at  $(1, 1)$ .
  - (c) Find the  $x$ -intercept of the tangent line to  $f(x)$  at  $x = a$ . Sketch  $f(x)$  and the tangent line.
  - (d) Find the  $y$ -intercept of the tangent line to  $f(x)$  at  $x = a$ .
  - (e) Find the  $y$ -intercept of the normal line to  $f(x)$  at  $x = a$ .
  - (f) What are the closest points on the graph of  $f(x)$  to the point  $(0, b)$ ?
  - (g) Which points on the graph of  $f(x)$  have a tangent line that pass through  $(0, -9)$ ?
5. Find equations of all lines having slope  $-1$  that are tangent to the curve  $y = \frac{1}{x-1}$ .

6. (a) Find all points on the graph of  $y = x^2$  whose tangent lines pass through the point  $(5, 0)$ .  
(b) Show that no line tangent to the graph of  $f(x) = x + \frac{1}{x}$  passes through the origin.
7. (a) For which values of  $x$  are the tangent lines to the parabola  $y = x^2$  and  $y = -x^2$  perpendicular?  
(b) Each point on the negative  $y$ -axis is a point of intersection of two lines tangent to the parabola  $y = x^2$ . At which points are these tangent lines perpendicular?
8. Suppose that  $f'(x) < 0$  for all real numbers  $x$ . What can you say about the minimum and maximum values of  $f$  on the interval  $[a, b]$ ?
9. Show that if  $0 \leq f(x) \leq 1$  for all  $0 \leq x \leq 1$  and  $f(x)$  is a continuous function, then there is some number  $a$  with  $0 \leq a \leq 1$  so that  $f(a) = a$ . (We then say that the function has a fixed point.)