

1. For each of the four cases below, sketch a graph of some function that satisfies the stated condition:

- (a) $\lim_{x \rightarrow 2} f(x) = 3$ and $f(2) = 4$
- (b) $\lim_{x \rightarrow 0} f(x)$ does not exist and $|f(x)| < 2$ for all x .
- (c) $\lim_{x \rightarrow 0} f(x) = f(0) + 1$
- (d) $\lim_{x \rightarrow \infty} f(x) = 1$ and $\lim_{x \rightarrow -\infty} f(x) = -1$

2. Recall that a function $f(x)$ is continuous at $x = a$ if $\lim_{x \rightarrow a} f(x) = f(a)$. Another definition you need is: A function $f(x)$ is continuous if it is continuous at every point. There are at least four ways for a function to fail to be continuous at a point $x = a$. Give examples of functions which fail to be continuous, including the point where continuity fails.

3. For each of the four following conditions, construct the desired example:

- (a) Find a continuous function $f(x)$ such that $f(-1) = f(2) = f(5) = 0$ and $f(0) \neq 0$.
- (b) Find a function that is not continuous at $x = 0$ but is continuous at any other x .
- (c) Find a function that is continuous at $x = 0$ but is not continuous at any other x .
- (d) Find a function that is continuous at each irrational number and not continuous at any rational numbers.

4. Which value(s) of a make the function $f(x)$ continuous?

$$f(x) = \begin{cases} ax^2 & x \geq 2 \\ \frac{a^2}{4}x^2 + 3 & x < 2 \end{cases}$$

5. Early in 1492, Cristóbol Colón was commissioned by King Ferdinand and Queen Isabella of Spain to journey west to reach the Orient. On September 6, 1492, Columbus (Colón's name in Latin) left the Canary Islands to make history. Not knowing exactly which direction he should head, Columbus manages to get his ship moving east or west according to the function

$$x(t) = (t - 36)^3(100 - t).$$

We pick up the action at $t = 0$ and follow him as $t \rightarrow \infty$. (Note that t is in days, $x(t)$ is in meters, and we ignore the reality that Columbus sailed southwestward, not just east and west.)

- (a) Assuming September 6, 1492 is the day we start watching, what is the position of the Canary Islands?
- (b) Since we are given that Columbus is initially traveling west, what is your sign convention?
- (c) For which t 's is Columbus' ship stopped?
- (d) When is he moving west? east?
- (e) There is an island at $x = 0$. How many times does Columbus go past this island? When?
- (f) When did Columbus stop at this island, called Guanahani by its natives, which he claimed for Spain and renamed San Salvador? From this, can you calculate the date of this renowned stop?
- (g) At $t = 67\frac{3}{4}$ days, Columbus realizes that his second-in-command, Pinzón, and the Pinta had vanished before a strong east wind. At this point, Columbus thought that it would be a good idea to go east and find his friend. When (for which time t) does the ship begin to slow down? What was the reaction time from seeing this to slowing down? How far did he travel in this time?
- (h) For which t is he

{	speeding up in the westward direction?
	slowing down in the westward direction?
	speeding up in the eastward direction?
	slowing down in the eastward direction?
- (i) What is his position when he finally starts going east toward the island of Haiti which he named Española (Hispaniola in Latin)?
- (j) How many days elapse before Columbus sets his sights upon San Salvador once again?
- (k) Columbus is now headed back to Spain. At what time t on his way to Spain are we sure that Columbus has passed the Canary Islands?
- (l) How can we alter our function $x(t) = (t - 36)^3(100 - t)$ so that Columbus is allowed to stop at Española and board the Niña since the Santa Maria ran aground at Española and was a total loss? Furthermore, how can we ensure (by altering the function $x(t)$) that Columbus eventually stops as $t \rightarrow \infty$? Keep in mind that after his stop in Haiti, Columbus must travel east.