
Complete the following problems. Show all work to receive full credit.

1. Find $\frac{d}{dx} \left(\frac{(3x+2)^4}{2x-11} \right)$

$$= \frac{4(3x+2)^3 \cdot 3(2x-11) - 2 \cdot (3x+2)^4}{(2x-11)^2}$$
$$= \frac{12(3x+2)^3(2x-11) - 2(3x+2)^4}{(2x-11)^2}$$

2. Find $\frac{d}{dx} ((1-x^2)\sec(2x))$

$$= -2x \sec(2x) + \sec(2x) \tan(2x) \cdot 2(1-x^2)$$
$$= -2x \sec(2x) + 2(1-x^2) \sec(2x) \tan(2x)$$

3. Find $f'(x)$ when $f(x) = x^2 \sin 2 + x^e + x^\pi + \pi^e$.

$$= 2x \sin 2 + ex^{e-1} + \pi x^{\pi-1}$$

4. Write the equation of the line perpendicular to the tangent line of $f(x) = 2x \cos x$ at $x = 0$.
The line will go through $(0, f(0))$. Let's find $f(0)$ first:

$$f(0) = 2 \cdot 0 \cos 0 = 0$$

So the line will go through $(0, 0)$

The slope is from the derivative;

$$f'(x) = 2 \cos x - 2x \sin x$$

At $x = 0$ this is

$$f'(0) = m = 2 \cos 0 - 2 \cdot 0 \sin 0$$
$$= 2$$

For the line perpendicular to the tangent line (called the normal line), the slope should be $m = -\frac{1}{2}$. So the equation of the line is

$$y = -\frac{1}{2}x$$

5. Consider the function $f(x) = \begin{cases} 3x^2 + 2x & x \geq 3 \\ 2x + 1 & 1 < x < 3 \\ x^2 + 2 & x < 1 \\ c & x = 1 \end{cases}$.

Find c in the above definition to make the function continuous at $x = 1$.

For this to exist, we need

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$$

Let's look at the limits first:

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 + 2 = 1^2 + 2 = 3$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 2x + 1 = 2(1) + 1 = 3$$

Therefore to make the function continuous, we need the function value, denoted by c to be the same as these limits, so $c = 3$.