Math 142
Spring, 2005 Name: **Solutions**
Quiz 6 - §§1.5. 2.1 (the beginning)

Complete the following problems. Show all work to receive full credit.

1. Find the **slope** of the tangent line to the curve $y = \sqrt{x+1}$ at the point $(8, 3)$.

$$
\begin{aligned}
m &= \lim_{h\to 0} \frac{f(8+h) - f(8)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{8+h+1} - 3}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{9+h} - 3}{h} \\
&= \lim_{h\to 0} \left(\frac{\sqrt{9+h} - 3}{h}\right)\left(\frac{\sqrt{9+h} + 3}{\sqrt{9+h} + 3}\right) \\
&= \lim_{h\to 0} \frac{9+h-9}{h\left(\sqrt{9+h}+3\right)} \\
&= \lim_{h\to 0} \frac{h}{h\left(\sqrt{9+h}+3\right)} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{9+h}+3} \\
&= \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}
\end{aligned}
$$

2. Find the **equation** of the tangent line to the curve $y = \sqrt{x+1}$ at the point $(8, 3)$. (**Hint:** Use your answer to problem 1.)

$$y - 3 = \frac{1}{6}(x - 8)$$

3. Find the derivative of the function $f(x) = \dfrac{1}{x^2}$.

$$
\begin{aligned}
&= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \\
&= \lim_{h\to 0} \frac{\frac{x^2}{x^2(x+h)^2} - \frac{(x+h)^2}{x^2(x+h)^2}}{h}
\end{aligned}
$$

$$= \lim_{h \to 0} \frac{\frac{x^2 - (x+h)^2}{x^2(x+h)^2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x^2 - (x^2 - 2xh + h^2)}{x^2(x+h)^2}}{h}$$

$$= \lim_{h \to 0} \frac{x^2 - x^2 - 2xh + h^2}{x^2(x+h)^2} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-2xh + h^2}{x^2 h(x+h)^2}$$

$$= \lim_{h \to 0} \frac{h(-2x + h)}{x^2 h(x+h)^2}$$

$$= \lim_{h \to 0} \frac{-2x + h}{x^2(x+h)^2}$$

$$= \frac{-2x}{x^2(x^2)}$$

$$f'(x) = \frac{-2}{x^3}$$