Math 142
Spring, 2005 Name: **Solutions**
Quiz 5 - §1.4

Complete the following problems. Show all work to receive full credit.

1. Where is the function $f(x) = \dfrac{1}{x-2} - 3x$ continuous? Explain your answer.

   This is the difference of functions, so it is continuous whenever each piece is continuous. Since the $3x$ *is always continuous (it is a polynomial), the only part that could cause trouble is the* $\dfrac{1}{x-2}$ *part. Since this is a rational function, it is continuous on its domain, so when* $x \neq 2$. *Therefore this function is continuous everywhere except when* $x = 2$.

2. Where is the function $f(x) = \sin(x - \sin x)$ continuous? Explain your answer.

   This is a composition of functions. The inside function is $x - \sin x$, *which is the difference of continuous functions, and is therefore continuous. The outside function is* $\sin x$ *which is also continuous. This is therefore the composition of continuous functions, and so is continuous everywhere.*

3. Examine the function $f(x) = x^3 - 8x + 10$. Explain why there is at least one value of c so that $f(c) = \pi$.

   We want to use the Intermediate Value Theorem. Since our function $f(x) = x^3 - 8x + 10$ *is a polynomial, it is continuous. Since we are interested in having an output of* π, *we just need to find two points - one where the output is less than* π *and one where it is greater than* π. *Then IVT will say that since the function is continuous, all of the points between those two, including* π *must be outputs also.*

$$f(0) = 0^3 - 8(0) + 10 = 10 > \pi$$

$$f(1) = 1^3 - 8(1) + 10 = 1 - 8 + 10 = 11 - 8 = 3 < \pi$$

   Therefore the function must have an output of π *somewhere between* $x = 0$ *and* $x = 1$,