Math 142
Fall 2007 Name: **Solutions**
Quiz 5 - §§2.6, 2.7, 3.1

Complete the following problems. Show all work to receive full credit.

1. Explain why the equation $\cos x = x$ has at least one solution.

   Let's look at $f(x) = \cos x - x$. This function is continuous since $\cos x$ is continuous and x is continuous. Therefore, we can apply the Intermediate Value Theorem. We want to show that $f(x) = \cos x - x$ can be 0.

   $$f(0) = \cos 0 - 0 = 1 - 0 = 1 > 0$$

   $$f(\pi) = \cos \pi - \pi = -1 - \pi < 0$$

   Therefore, $f(x) = 0$ somewhere between 0 and π.

2. Use the **definition of the derivative** to find the derivative of $f(x) = \dfrac{1-x}{2x}$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1-(x+h)}{2(x+h)} - \frac{1-x}{2x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1-x-h)}{2x+2h} - \frac{1-x}{2x}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\left(\frac{1-x-h}{2x+2h}\right)\left(\frac{2x}{2x}\right) - \left(\frac{1-x}{2x}\right)\left(\frac{2x+2h}{2x+2h}\right)\right)$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{2x - 2x^2 - 2xh}{(2x+2h)(2x)} - \frac{2x + 2h - 2x^2 - 2xh}{2x(2x+2h)}\right)$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{2x - 2x^2 - 2xh - 2x - 2h + 2x^2 + 2xh}{(2x+2h)(2x)}\right)$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{-2h}{(2x+2h)(2x)}\right)$$

$$= \lim_{h \to 0} \frac{-2}{(2x+2h)(2x)}$$

$$= \frac{-2}{(2x)^2}$$

There are more problems on the reverse.

3. Use the following axes. Sketch a graph of the derivative of the given function.