Math 142
Fall 2007
Quiz 3 - §§2.2, 2.3, 2.4

Name: **Solutions**

Complete the following problems. Show all work to receive full credit.

1. Calculate the following limits:

   (a) $\lim_{x \to 5} \frac{x-5}{x^2-25}$

$$
\begin{aligned}
&= \lim_{x \to 5} \frac{x-5}{(x-5)(x+5)} \\
&= \lim_{x \to 5} \frac{1}{x+5} \\
&= \frac{1}{10}
\end{aligned}
$$

   (b) $\lim_{x \to 2} \frac{\sqrt{x^2+12}-4}{x-2}$

$$
\begin{aligned}
&= \lim_{x \to 2} \left(\frac{\sqrt{x^2+12}-4}{x-2} \right) \cdot \left(\frac{\sqrt{x^2+12}+4}{\sqrt{x^2+12}+4} \right) \\
&= \lim_{x \to 2} \frac{x^2+12-16}{(x-2)(\sqrt{x^2+12}+4)} \\
&= \lim_{x \to 2} \frac{x^2-4}{(x-2)(\sqrt{x^2+12}+4)} \\
&= \lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)(\sqrt{x^2+12}+4)} \\
&= \lim_{x \to 2} \frac{x+2}{\sqrt{x^2+12}+4} \\
&= \frac{2+2}{\sqrt{4+12}+4} \\
&= \frac{4}{\sqrt{16}+4} \\
&= \frac{4}{4+4} \\
&= \frac{4}{8} \\
&= \frac{1}{2}
\end{aligned}
$$

(c) $\lim_{x\to 0} \dfrac{\tan 2x}{x}$

$$\begin{aligned}
&= \lim_{x\to 0} \frac{\sin 2x}{\cos 2x} \cdot \frac{1}{x} \\
&= \lim_{x\to 0} \frac{1}{\cos 2x} \cdot \frac{\sin 2x}{x} \\
&= \lim_{x\to 0} \frac{1}{\cos 2x} \cdot \frac{2\sin 2x}{2x} \\
&= \lim_{x\to 0} \frac{1}{\cos 2x} \cdot \lim_{x\to 0} 2\,\frac{\sin 2x}{2x} \\
&= \frac{1}{\cos 0} \cdot 2 \cdot 1 \\
&= \frac{1}{1} \cdot 2 \\
&= 2
\end{aligned}$$

2. You know that $\lim_{x\to 4} x^2 - 5 = 11$. Find the δ that corresponds to $\epsilon = 1$ in the precise definition of a limit for this function near $a = 4$.

$$11 - 1 < x^2 - 5 < 11 + 1$$
$$10 < x^2 - 5 < 12$$
$$15 < x^2 < 17$$
$$\sqrt{15} < x < \sqrt{17}$$
$$\sqrt{15} - 4 < x - 4 < \sqrt{17} - 4$$

$\sqrt{15} - 4 \approx 3.87 - 4 \approx -.127$ and $\sqrt{17} - 4 \approx 4.12 - 4 \approx .12$

Therefore, we chose $\delta = 0.12$. This will satisfy our requirement.