Math 142
Spring, 2006
Quiz 25 - §§5.5, 5.6

Name: **Solutions**

Complete the following problems. Show all work to receive full credit.

1. $\displaystyle\int \frac{6\cos t}{(2+\sin t)^3}\,dt$

$$u = 2+\sin t \qquad du = \cos t\,dt$$

$$\begin{aligned}\int \frac{6\cos t}{(2+\sin t)^3}\,dt &= \int \frac{6}{u^3}du \\ &= \int 6u^{-3}\,du \\ &= -3u^{-2}+C \\ &= \frac{-3}{(2+\sin t)^2}+C\end{aligned}$$

2. $\displaystyle\int \sqrt{\frac{x-1}{x^5}}\,dx$

$$\begin{aligned}\int \sqrt{\frac{x-1}{x^5}}\,dx &= \int \sqrt{\frac{1}{x^4}\frac{x-1}{x}}\,dx \\ &= \int \frac{1}{x^2}\sqrt{\frac{x-1}{x}}\,dx \\ &= \int \frac{1}{x^2}\sqrt{1-\frac{1}{x}}\,dx\end{aligned}$$

$$u = 1-\frac{1}{x} \qquad du = \frac{1}{x^2}dx$$

$$\begin{aligned}\int \frac{1}{x^2}\sqrt{1-\frac{1}{x}}\,dx &= \int \sqrt{u}\,du \\ &= \frac{2}{3}u^{\frac{3}{2}}+C \\ &= \frac{2}{3}\left(1-\frac{1}{x}\right)^{\frac{3}{2}}+C\end{aligned}$$

3. $\displaystyle\int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta \; d\theta$

$$u = \cos 2\theta \qquad du = -2\sin 2\theta \; d\theta$$

$$\begin{aligned}
\int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta \; d\theta &= \int_0^{\frac{\pi}{6}} (\cos 2\theta)^{-3} \sin 2\theta \; d\theta \\
&= \int_{\theta=0}^{\theta=\frac{\pi}{6}} \frac{-1}{2} u^{-3} \; du \\
&= \frac{1}{4} u^{-2} \Big|_{\theta=0}^{\theta=\frac{\pi}{6}} \\
&= \frac{1}{4} (\cos 2\theta)^{-2} \Big|_{\theta=0}^{\theta=\frac{\pi}{6}} \\
&= \frac{1}{4} \left(\cos \frac{\pi}{3}\right)^{-2} - \frac{1}{4} (\cos 0)^{-2} \\
&= \frac{1}{4} \left(\frac{\sqrt{3}}{2}\right)^{-2} - \frac{1}{4} \\
&= \frac{1}{4} \left(\frac{2}{\sqrt{3}}\right)^{2} - \frac{1}{4} \\
&= \frac{1}{4} \cdot \frac{4}{3} - \frac{1}{4} \\
&= \frac{1}{3} - \frac{1}{4} \\
&= \frac{1}{12}
\end{aligned}$$