Math 142
Spring, 2006 Name: **Solutions**
Quiz 23 - §5.3

Complete the following problems. Show all work to receive full credit.

1. Let $f(x)$ be a function defined on a closed interval $[a, b]$. We say that a number I is the definite integral of f over $[a, b]$ if:

$$I = \lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\Delta x_k$$

where $\Delta x_k = x_k - x_{k-1}$, no matter how the c_k's are chosen in the $k-$th subinterval.

You may need the following facts:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

2. Use the definition of the definite integral to calculate:

$$\int_0^{\sqrt{2}} \left(x - \sqrt{2}\right)\, dx$$

$$\Delta x = \frac{\sqrt{2} - 0}{n} = \frac{\sqrt{2}}{n} \qquad c_k = 0 + k \cdot \frac{\sqrt{2}}{n} = k \cdot \frac{\sqrt{2}}{n}$$

$$\begin{aligned}
\int_0^{\sqrt{2}} \left(x - \sqrt{2}\right)\, dx &= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(k \cdot \frac{\sqrt{2}}{n}\right) \cdot \frac{\sqrt{2}}{n} \\
&= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{\sqrt{2}\,k}{n} - \sqrt{2}\right) \cdot \frac{\sqrt{2}}{n} \\
&= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{2k}{n^2} - \frac{2}{n} \\
&= \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{2}{n^2}k - \sum_{k=1}^{n} \frac{2}{n}\right) \\
&= \lim_{n\to\infty} \frac{2}{n^2}\frac{n(n+1)}{2} - \frac{2}{n} \cdot n \\
&= \lim_{n\to\infty} \frac{n(n+1)}{n^2} - 2 \\
&= 1 - 2 \\
&= -1
\end{aligned}$$