
Complete the following problems. Show all work to receive full credit.

1. Find the requested limit:

(a) $\lim_{x \rightarrow \infty} \frac{\log_2 x}{\log_3(x+3)}$

$$\begin{aligned} &=^{LH} \lim_{x \rightarrow \infty} \frac{\frac{1}{x \ln 2}}{\frac{1}{(x+3) \ln 3}} \\ &= \lim_{x \rightarrow \infty} \frac{(x+3) \ln 3}{x \ln 2} \\ &= \lim_{x \rightarrow \infty} \frac{x \ln 3 + 3 \ln 3}{x \ln 2} \\ &=^{LH} \lim_{x \rightarrow \infty} \frac{\ln 3}{\ln 2} \\ &= \frac{\ln 3}{\ln 2} \end{aligned}$$

(b) $\lim_{x \rightarrow 0} \frac{\sin 7x}{\tan 11x}$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{7 \cos 7x}{11 \sec^2 11x} \\ &= \lim_{x \rightarrow 0} \frac{7}{11} \cos 7x \cos^2 11x \\ &= \frac{7}{11} \cdot 1 \cdot 1 = \frac{7}{11} \end{aligned}$$

(c) $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}}$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} e^{\ln f(x)} = e^{\lim_{x \rightarrow 0} \ln f(x)}$$

$$\lim_{x \rightarrow 0} \ln f(x) = \lim_{x \rightarrow 0} \frac{1}{x} \ln(1+x)$$

$$= \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x}$$

$$=^{LH} \lim_{x \rightarrow 0} \frac{\frac{1}{1+x}}{1}$$

$$= \lim_{x \rightarrow 0} \frac{1}{1+x} = \frac{1}{1} = 1$$

Therefore the desired limit is

$$\lim_{x \rightarrow 0} f(x) = e^1 = e$$