Math 142
Spring, 2006 Name: **Solutions**
Quiz 21 - §4.6, 4.8

Complete the following problems. Show all work to receive full credit.

1. Calculate the following limits:

   (a) $\lim_{x\to 0^+} \frac{x2^x}{2^x - 1}$
   This is an indeterminate form of type $\frac{0}{0}$

$$=^{LH} \lim_{x\to 0^+} \frac{2^x + x2^x \ln 2}{2^x \ln 2}$$
$$= \lim_{x\to 0^+} \frac{1 + x\ln 2}{\ln 2}$$
$$= \frac{1}{2}$$

   (b) $\lim_{x\to\infty} (\ln x)^{\frac{1}{x}}$
   We look at

$$y = (\ln x)^{\frac{1}{x}}$$
$$\ln y = \frac{1}{x}\ln(\ln x)$$
$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(\ln x)}{x}$$

   This is indeterminate of type $\frac{\infty}{\infty}$, so we use L'Hopital's rule:

$$\lim_{x\to\infty} \frac{\ln(\ln x)}{x} = \lim_{x\to\infty} \frac{\frac{1}{x\ln x}}{1}$$
$$= \lim_{x\to\infty} \frac{1}{x\ln x} = 0$$

   Therefore $\lim_{x\to\infty} y = \lim_{x\to\infty} e^0 = 1$

2. Calculate the following indefinite integrals:

   (a) $\int x^{-3}(x+1)\,dx$

$$= \int x^{-2} + x^{-3} dx$$
$$= -x^{-1} - \frac{1}{2}x^{-2} + C$$

   (b) $\int e^{-x} + 4^x + \frac{2}{\sqrt{1-x^2}} dx$

$$= -e^{-x} + \frac{1}{\ln 4}4^x + 2\sin^{-1} x + C$$