
Complete the following problems. Show all work to receive full credit.

1. Find the equilibria values and state whether they are stable or unstable for the differential equation:

$$\frac{dy}{dx} = (y - 1)(y - 2)(y - 3)$$

The equilibria occur when $\frac{dy}{dx} = 0$, so where $y = 1$, $y = 2$ and $y = 3$.

Checking points in the interval, we have:

$$\begin{aligned}y'(0) &= -1 \cdot -2 \cdot -3 < 0 \\y'\left(\frac{3}{2}\right) &= \frac{1}{2} \cdot -\frac{1}{2} \cdot -\frac{3}{2} > 0 \\y'\left(\frac{5}{2}\right) &= \frac{3}{2} \cdot \frac{1}{2} \cdot -\frac{1}{2} < 0 \\y'(4) &= 3 \cdot 2 \cdot 1 > 0\end{aligned}$$

Therefore $y = 1$ is a unstable equilibrium, $y = -2$ is a stable equilibrium, and $y = 3$ is an unstable equilibrium.

2. Use Newton's method to estimate the zero of the function $f(x) = 2x - x^2 + 1$ near $x_0 = 2$. Estimate to x_2 .

$$\begin{aligned}x_0 &= 2 \\f(x) &= 2x - x^2 + 1 \\f'(x) &= 2 - 2x \\f(x_0) &= f(2) = 4 - 4 + 1 = 1 \\f'(x_0) &= f'(2) = 2 - 2(2) = -2 \\x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{1}{-2} = 2 + \frac{1}{2} = \frac{5}{2} \\f(x_1) &= f\left(\frac{5}{2}\right) = 2\left(\frac{5}{2}\right) - \left(\frac{5}{2}\right)^2 + 1 = 5 - \frac{25}{4} + 1 = -\frac{1}{4} \\f'(x_1) &= f'\left(\frac{5}{2}\right) = 2 - 2 \cdot \frac{5}{2} = 2 - 5 = -3 \\x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{5}{2} - \frac{-\frac{1}{4}}{-3} = \frac{29}{12}\end{aligned}$$