Math 142
Spring, 2005 Name:
Quiz 20 - §3.3

Complete the following problems. Show all work to receive full credit.

1. For all problems, consider the function $y = 2x^4 - 4x^2 + 1$

   (a) Find the critical points of the function.

$$f'(x) = 8x^3 - 8x$$
$$f'(x) = 8x(x^2 - 1)$$
$$f'(x) = 0 \text{ when } 8x(x^2 - 1) = 0$$
$$x = 0 \text{ or } x = 1 \text{ or } x = -1$$

   (b) Find the intervals where the function is decreasing.
   We want $f'(x) < 0$. Use the critical points from the first part to check intervals:

$$f'(-2) = -48 < 0$$
$$f'\left(-\frac{1}{2}\right) = 3 > 0$$
$$f'\left(\frac{1}{2}\right) = -3 < 0$$
$$f'(2) = 48 > 0$$

   So the only intervals where it is increasing is $(0, 1)$ and $(-\infty, -1)$.

   (c) Find the possible points of inflection.

$$f''(x) = 24x^2 - 8$$
$$f''(x) = 0 \text{ when } 24x^2 - 8 = 0$$
$$24x^2 = 8 \Rightarrow x^2 = \frac{1}{3} \Rightarrow x = \pm\sqrt{\frac{1}{3}}$$

   (d) Find the intervals where the function is concave up.
   Using the points of inflection to check intervals, we want to find out where $f''(x) > 0$:

$$f''(-1) = 24(-1)^2 - 8 = 16 > 0$$
$$f''(0) = 24(0)^2 - 8 = -8 < 0$$
$$f''(1) = 24(1)^2 - 8 > 0$$

   Therefore the function is concave up on the intervals $\left(-\infty, -\sqrt{\frac{1}{3}}\right)$ and $\left(\sqrt{\frac{1}{3}}, \infty\right)$