Math 142
Spring, 2005 Name: **Solutions**
Quiz 19 - §3.5

Complete the following problems. Show all work to receive full credit.

1. What is the smallest perimeter possible for a rectangle whose area is 16 in^2?

$$A = lw = 16$$
$$P = 2l + 2w$$
$$l = \frac{16}{w}$$
$$P(w) = 2\left(\frac{16}{w}\right) + 2w$$
$$P(w) = 32w^{-1} + 2w$$
$$P'(w) = -32w^{-2} + 2$$
$$P'(w) = 0 \text{ when } \frac{-32}{w^2} + 2 = 0$$
$$\frac{-32}{w^2} = -2$$
$$-32 = -2w^2$$
$$16 = w^2$$
$$w = \pm 4$$

To check that $w = 4$ gives a minimum, use the second derivative:

$$P'' = 64w^{-3} = \frac{64}{w^3}$$

At the critical point, $w = 4$, this is

$$P''(4) = \frac{64}{4^3} > 0$$

Therefore the function is concave up, so the critical point is a minimum. Therefore the smallest perimeter possible is

$$P(4) = 2\left(\frac{16}{4}\right) + 2(4)$$
$$= 2(4) + 8 = 8 + 8 = 16$$

2. A 216 m^2 rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence?

   Let's assume that the sides are called l and w. Assume that the fence is parallel to the l side. Then we know:

$$lw = 216 \Rightarrow l = \frac{216}{w}$$
$$P = 2w + 3l$$
$$P(w) = 2w + 3\left(\frac{216}{w}\right)$$
$$P(w) = 2w - 648w^{-1}$$
$$P'(w) = 2 - 648w^{-2}$$
$$P'(w) = 0 \text{ when } 2 - \frac{648}{w^2} = 0$$
$$2 = \frac{648}{w^2}$$
$$2w^2 = 648$$
$$w^2 = 324$$
$$w = \pm 18$$

   To make sure that this is a minimum, use the second derivative:

$$P''(w) = \frac{1248}{w^3}$$

   At the critical point, this is

$$P''(18) = \frac{1248}{18^3} > 0$$

   Therefore this is a minimum.

   The dimensions of the outer rectangle should then be 18×12.