Math 142
Spring, 2006 Name: **Solutions**
Quiz 18 - §§4.1, 4.2

Complete the following problems. Show all work to receive full credit.

1. Find the absolute maximum and absolute minimum for the function $f(x) = \dfrac{1}{x} - \ln x$ on the interval $\dfrac{1}{2} \leq x \leq 4$.

$$f'(x) = -\frac{1}{x^2} - \frac{1}{x} = -\frac{1}{x^2} - \frac{x}{x^2} = \frac{-1-x}{x^2}$$

This function dne at $x = 0$ (which is not in the required interval) and is zero where

$$-1 - x = 0$$

$$x = -1$$

which is also not in the required interval. We don't have these points to check, so we need to check the endpoints:

$$f\left(\frac{1}{2}\right) = 2 - \ln\frac{1}{2} \approx 2.6931$$

$$f(4) = \frac{1}{4} - \ln 4 \approx -1.136$$

Therefore the absolute minimum of $\frac{1}{4} - \ln 4$ occurs at $x = 4$ and the absolute maximum of $2 - \ln\frac{1}{2}$ occurs at $x = \frac{1}{2}$

2. Does the function $f(x) = \sqrt{x(1-x)}$ satisfy the hypothesis of the Mean Value Theorem on the interval $[0, 1]$? Show all work. If it does, find the c that MVT guarantees.

This function is continuous on $[0, 1]$ and differentiable on $(0, 1)$, so the MVT says there is a $c \in (0, 1)$ so that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0} = 0$$

$$f'(c) = \frac{1}{2}\left(x(1-x)\right)^{-\frac{1}{2}} \cdot (1 - 2x)$$

We want this to be 0, so

$$\frac{1-2x}{2\sqrt{x-x^2}} = 0$$

where the numerator is 0:

$$1 - 2x = 0$$

$$1 = 2x$$

$$x = \frac{1}{2}$$

which is in the correct interval.