Math 142
Spring, 2005 Name: **Solutions**
Quiz 17 - §§3.2, 3.3

Complete the following problems. Show all work to receive full credit.

Notice that I have changed the first problem - the way it was originally on the quiz, there was no minimum because of the vertical asymptote at $x = 0$

1. Find the absolute extrema of the function $f(x) = -\frac{1}{x^2}$ on the interval $0.5 \leq x \leq 2$.

$$f'(x) = 2x^{-3} = \frac{2}{x^3}$$

Therefore the only critical value if where the derivative is undefined, i.e. at $x = 0$. We check this critical point and the end points:

$$f(0) \text{ is undefined and not a max or min}$$
$$f(-0.5) = -\frac{1}{\left(-\frac{1}{2}\right)^2} = -\frac{1}{\frac{1}{4}} = -4$$
$$f(2) = -\frac{1}{2^2} = -\frac{1}{4}$$

Therefore the absolute maximum occurs at $\left(2, -\frac{1}{4}\right)$ and the absolute minimum occurs at $(0.5, 4)$.

2. (a) Show that the function $f(x) = x^2 + 2x - 1$ satisfies the hypotheses of the Mean Value Theorem on $[0, 1]$.

We need to show that the function is continuous and differentiable. Since the function is a polynomial, it is continuous. Also, the derivative is

$$f'(x) = 2x + 2$$

so it is differentiable. Therefore it satisfies the hypotheses of the Mean Value Theorem

(b) Find the value of c in (a, b) that satisfies the equation $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ on $[0, 1]$.

As noted, $f'(x) = 2x + 2$. We want to see where this is equal to

$$\frac{f(1) - f(0)}{1 - 0} = \frac{1^2 + 2(1) - 1 - (0^2 + 2(0) - 1)}{1} = \frac{2 + 1}{1} = 3$$

$$2x + 2 = 3$$
$$2x = 1$$
$$x = \frac{1}{2}$$

which is in the correct interval.