
Complete the following problems. Show all work to receive full credit.

1. Find $\frac{d}{dx} (\csc^{-1}(x^2 + 1))$

$$\begin{aligned} &= -\frac{1}{|x^2 + 1| \sqrt{(x^2 - 1)^2 - 1}} \cdot 2x \\ &= -\frac{2x}{|x^2 + 1| \sqrt{x^4 - 2x^2 + 1 - 1}} \\ &= -\frac{2x}{|x^2 + 1| \sqrt{x^4 - 2x^2}} \end{aligned}$$

2. Find $\frac{d}{dt} (\sin^{-1} \sqrt{2}t)$

$$= \frac{1}{\sqrt{1 - (\sqrt{2}t)^2}} \cdot \sqrt{2} = \frac{\sqrt{2}}{\sqrt{1 - 2t^2}}$$

3. Find $\frac{d}{dx} (\tan^{-1} \sqrt{x^2 - 1} + x \sin^{-1} x)$

$$\begin{aligned} &= \frac{1}{1 + (\sqrt{x^2 - 1})^2} \cdot \frac{1}{2}(x^2 - 1)^{-\frac{1}{2}} \cdot 2x + \sin^{-1} x + x \frac{1}{\sqrt{1 - x^2}} \\ &= \frac{x}{(1 + x^2 - 1)\sqrt{x^2 - 1}} + \sin^{-1} x + x \frac{1}{\sqrt{1 - x^2}} \\ &= \frac{x}{x^2 \sqrt{x^2 - 1}} + \sin^{-1} x + x \frac{1}{\sqrt{1 - x^2}} \\ &= \frac{1}{x \sqrt{x^2 - 1}} + \sin^{-1} x + x \frac{1}{\sqrt{1 - x^2}} \end{aligned}$$

4. Find $(f^{-1})'(x)$ at $x = 1$ if $f(x) = x^3 - 3x^2 + 2x + 1$.

$$f'(x) = 3x^2 - 6x + 2$$

$$f^{-1}(1) = t \text{ means } f(t) = 1$$

This is true when $t = 0$ since $f(0) = 0^3 + 3(0)^2 + 2(0) + 1$ So, we have

$$(f^{-1})'(1) = \frac{1}{f'(0)} = \frac{1}{3(0)^2 - 6(0) + 2} = \frac{1}{2}$$