Math 142
Fall 2007 Name: **Solutions**
Quiz 12 - §§5.1, 5.2

Complete the following problems. Show all work to receive full credit.

1. Approximate the area under the curve $f(x) = 4 - x^2$ between $x = -2$ and $x = 2$ using four subintervals of equal width and right-hand endpoints.

Interval	Right hand endpoint	$f(\text{RHE})$	Δx	Area
$(-2, -1)$	-1	0	1	0
$(-1, 0)$	0	4	1	4
$(0, 1)$	1	3	1	3
$(1, 2)$	2	0	1	0
Total				7

2. Calculate $\sum_{k=1}^{5} \sin k\pi$.

$$= \sin\pi + \sin 2\pi + \sin 3\pi + \sin 4\pi + s \in 5\pi = 0 + 0 + 0 + 0 + 0 = 0$$

3. Find $\int_2^4 x\ dx = \lim_{n\to\infty} \sum_{k=1}^{n} \left(2 + k\frac{2}{n}\right)\left(\frac{2}{n}\right)$ by calculating the limit of the sum.

$$\int_2^4 x\ dx = \lim_{n\to\infty} \sum_{k=1}^{n} \left(2 + k\frac{2}{n}\right)\left(\frac{2}{n}\right) = \lim_{n\to\infty} \frac{2}{n} \sum_{k=1}^{n} \left(2 + k\frac{2}{n}\right)$$
$$= \lim_{n\to\infty} \frac{2}{n}\left(2n + \frac{2}{n}\frac{n(n+1)}{2}\right)$$
$$= \lim_{n\to\infty} 4 + \frac{2(n+1)}{n}$$
$$= \lim_{n\to\infty} 4 + \frac{2n+2}{n}$$
$$=^{LH} 4 + \lim_{n\to\infty} \frac{2}{1}$$
$$= 4 + 2 = 6$$

<u>Bonus</u> Check your answer to number 3 by finding the area under the curve.

The area under the curve is the area under the big triangle - area under the small triangle.

Area of big triangle $= \frac{1}{2}(4)(4) = 8$

Area of small triangle $= \frac{1}{2}(2)(2) = 2$

Therefore the areas under the curve is $8 - 2 = 6$.