Math 142
Fall 2007
Quiz 11 - §§4.6, 4.8

Name: **Solutions**

Complete the following problems. Show all work to receive full credit.

1. Calculate the following limits:

(a) $\displaystyle\lim_{x\to 1}\frac{x-1}{\ln x-\sin\pi x}$

Notice that this is indeterminate of form $\frac{0}{0}$, so we can use l'Hopital's rule:

$$\begin{aligned}
&=^{LH}\lim_{x\to 1}\frac{1}{\frac{1}{x}-\pi\cos\pi x}\\
&=\lim_{x\to 1}\frac{1}{\frac{1-\pi x\cos\pi x}{x}}\\
&=\lim_{x\to 1}\frac{x}{1-\pi x\cos\pi x}\\
&=\frac{1}{1-\pi\cos\pi}\\
&=\frac{1}{1+\pi}
\end{aligned}$$

(b) $\displaystyle\lim_{x\to\infty}(\ln x)^{\frac{1}{x}}$

Notice that this is indeterminate of type ∞^∞. Let $y=(\ln x)^{\frac{1}{x}}$

$$\lim_{x\to\infty}y=\lim_{x\to\infty}e^{\ln y}=\lim_{x\to\infty}e^{\frac{1}{x}\ln(\ln x)}=e^{\lim_{x\to\infty}\frac{1}{x}\ln(\ln x)}$$

So we will examine the limit of the exponent:

$$\lim_{x\to\infty}\frac{\ln(\ln x)}{x}$$

This is of form $\frac{\infty}{\infty}$, so we use l'Hoptial's rule:

$$\begin{aligned}
&=^{LH}\lim_{x\to\infty}\frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1}\\
&=\lim_{x\to\infty}\frac{1}{x\ln x}\\
&=0
\end{aligned}$$

Now $e^0=1$, so

$$\lim_{x\to\infty}y=\lim_{x\to\infty}e^{\ln y}=\lim_{x\to\infty}e^{\frac{1}{x}\ln(\ln x)}=e^{\lim_{x\to\infty}\frac{1}{x}\ln(\ln x)}=e^0=1$$

2. Find the following indefinite integrals:

(a) $\int x^{-3}(x+1)\ dx$

$$= \int x^{-2} + x^{-3}\ dx$$
$$= -x^{-1} + \frac{1}{-2}x^{-2} + C$$
$$= -\frac{1}{x} - \frac{1}{2x^2} + C$$

(b) $\int \left(e^{-x} + 4^x\right)\ dx$

$$= -e^{-x} + \frac{1}{\ln 4}4^x + C$$

(c) $\int \left(\sin 2x - \csc^2 x\right)\ dx$

$$= \int 2\sin x \cos x - \csc^2 x\ dx$$
$$= \sin^2 x - \cot x + C$$

OR

$$\int \left(\sin 2x - \csc^2 x\right)\ dx = -\frac{1}{2}\cos 2x + \cot x + C$$