Math 142
Spring, 2005
Final Exam

Part I - Definitions and Theorems. State the requested definition or theorem. (2 points each)

1. State the definition of a function.
2. A function f is continuous at $x = a$ if
3. State the definition of the derivative, $f'(x)$.
4. State Rolle's Theorem.
5. State the Fundamental Theorem of Calculus, Part I.

Part II - Calculations. Show all work to receive credit. NO WORK = NO CREDIT.

6. (4 points each)Calculate the following limits:
   (a) $\lim_{x\to 3} \dfrac{x^2 + x - 12}{x - 3}$
   (b) $\lim_{x\to \infty} \dfrac{3x^2 - 4x + 5}{-5x + 7}$
   (c) $\lim_{x\to 0} \dfrac{\sin 7x}{3x}$
7. (6 points) Use the **definition of derivative** to find $f'(x)$ for $f(x) = \dfrac{1}{\sqrt{x+2}}$.
8. (5 points each) Compute the following derivatives:
   (a) $\dfrac{d}{dx}\left(x^{\sin x}\right)$
   (b) $\dfrac{d}{dx}\left(\sqrt{x + \dfrac{3x}{\cos x}}\right)$
   (c) $\dfrac{d}{dx}\left(e^x + x^3 + \ln x^2 + e^\pi + \sec x + \tan^{-1} x\right)$
9. (6 points) Find all points where the tangent line to the graph of the equation $g(x) = x^3 - 3x$ is perpendicular to the line $5y - 3x - 8 = 0$, or explain why this cannot occur.
10. (6 points) Find the equation of the line tangent to the curve $x + \sqrt{xy} = 6$ at the point $(4, 1)$.
11. (6 points) Find all values of x where $f(x) = \dfrac{-5x}{5x - 15}$ is concave up.

12. (6 points) For what value(s) of a will the function $f(x) = \begin{cases} e^x & x < 0 \\ a + x & x \geq 0 \end{cases}$ be continuous at $x = 0$? Explain.

13. Consider the function $y = x \sin x$ on the interval $[0, 2\pi]$.

    (a) (3 points) Find the average rate of change of the function over this interval.

    (b) (3 points) Find a place in this interval where the instantaneous rate of change of the function is equal to the average rate of change.

14. (3 points each) Calculate the following integrals:

    (a) $\displaystyle\int x^4 - \frac{7}{x} + \sin x \, dx$

    (b) $\displaystyle\int \frac{\tan^{-1} x}{1 + x^2} \, dx$

    (c) $\displaystyle\int_1^3 \frac{x^3 + 3x + 7}{\sqrt{x}} \, dx$

15. (3 points) Find the area under the curve $y = \sec x \tan x$ from $-\frac{\pi}{3}$ to 0.

16. (3 points each) Calculate the following:

    (a) $\displaystyle\frac{d}{dx}\left(\int_{\sin x}^{e^2} \frac{1}{x\sqrt{\ln x}} \, dx\right)$

    (b) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^7 x \, dx$

Part III - Applications. Show all work to receive credit. NO WORK= NO CREDIT.

17. (7 points) A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a double-strand electric fence. (Think about what that means!) With 800 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?

18. (7 points) A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 feet per second, how fast is the angle between the top of the ladder and the wall changing when the angle is $\frac{\pi}{4}$ radians?