Math 142
Spring, 2004
Final Exam

Part I - Definitions and Theorems State the requested definition or theorem. (2 points each)

1. A function f is continuous at $x = a$ if
2. State the definition of the derivative, $f'(x)$.
3. State the Intermediate Value Theorem.
4. State the Mean Value Theorem.
5. State the Extreme Value Theorem.
6. State the Fundamental Theorem of Calculus, Part II.

Part II - Calculations

Show all work to receive credit. NO WORK = NO CREDIT.

7. (5 points each)Calculate the following limits:
   (a) $\lim_{x\to -2} \dfrac{x^2 - 2x - 8}{x + 2}$
   (b) $\lim_{x\to \infty} \dfrac{3x^2 - 4x + 5}{-5x^2 + 7}$
   (c) $\lim_{x\to 0} \dfrac{\sin 5x}{4x}$
8. (6 points) Use the **definition of derivative** to find $f'(x)$ for $f(x) = \frac{1}{x+2}$.
9. (5 points each) Compute the following derivatives:
   (a) $\dfrac{d}{dx}\left(\left(\dfrac{4x}{x+1}\right)^{-2}\right)$
   (b) $\dfrac{d}{dx}\left(\sqrt{x + \sqrt{x}}\right)$
   (c) $\dfrac{d}{dx}\left((1 + x^2)\pi^x\right)$
   (d) $\dfrac{d}{dx}\left(2\,(\ln x)^{\frac{x}{2}}\right)$
10. (6 points) Find all points where the tangent line to the graph of the equation $e^{x+2y} = 1$ is parallel to the line $y = x + 7$.
11. (6 points) Find the equation of the line tangent to the curve $x + \sqrt{xy} = 6$ at the point $(4, 1)$.

12. (6 points) Find all intervals where the slope of the function $f(x) = x^3 + 3x^2 - 3x + 29$ is increasing.

13. (6 points) Find all intervals where the function $y = \dfrac{\ln x}{x}$ is concave down.

14. (6 points) Find all values of the constants m and b for which the function

$$f(x) = \begin{cases} \sin x & \text{if} \quad x < \pi \\ mx + b & \text{if} \quad x \geq \pi \end{cases}$$

is continuous at $x = \pi$.

15. Consider the function $y = x^{\frac{2}{3}}$ on the interval $[0, 1]$.

    (a) (3 points) Find the average rate of change of the function over this interval.

    (b) (3 points) Find a place in this interval where the instantaneous rate of change of the function is equal to the average rate of change.

16. (3 points each) Calculate the following indefinite integrals:

    (a) $\displaystyle\int (x^3 + 5x - 7)dx$

    (b) $\displaystyle\int \left(\frac{1}{2\sqrt{t}} - \frac{3}{t^4}\right) dt$

    (c) $\displaystyle\int \frac{1}{\sqrt{1 - x^2}} dx$

17. (6 points) Find the area under the curve $y = \sec x \tan x$ from $-\frac{\pi}{3}$ to 0.

18. (3 points each) Calculate the following:

    (a) $\displaystyle\int_e^{e^2} \frac{1}{x\sqrt{\ln x}} dx$

    (b) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{4}} z^{29} \csc^2 z \cot z dx$

Part III - Applications Show all work to receive credit. NO WORK= NO CREDIT.

19. ($\frac{20}{3}$ points) On our moon, the acceleration of gravity if 1.6 m/sec^2. If a rock is dropped into a crevasse, how fast will it be going just before it hits bottom 30 sec later.

20. ($\frac{20}{3}$ points) A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?

21. ($\frac{20}{3}$ points) The length l of a rectangle is decreasing at the rate of 2 cm/sec while the width w is increasing at the rate of 2 cm/sec. When $l = 12$and $w = 5$, find the rate of change of the area of the rectangle.