Math 142
Spring, 2005
Exam 2
Chapters 2 and 3

Part I - Definitions and Theorems State the requested definition or theorem.

1. (3 points) State Rolle's Theorem.
2. (3 points) State the Extreme Value Theorem.

   Part II - Calculations

   Show all work to receive credit. NO WORK = NO CREDIT.

3. Compute the following derivatives:
   (a) (7 points) Find $\frac{d}{dx}\left(e^{\pi}+\pi^{x}+e^{x}+\ln 2+\sqrt{\pi x}\right)$
   (b) (8 points) Find $\frac{dy}{dx}$ when $y = x^{\sin x}$
   (c) (4 points) Find $\frac{d}{dx}\left(\sin^{-1}(\tan(\pi x))\right)$
   (d) (5 points) Find $\frac{d}{dy}\left(5e^{1-3y}\ln(3y)\right)$
   (e) (8 points) Find $\frac{dy}{dx}$ for $y = \frac{x\sqrt{x^2+1}}{(x+1)^{\frac{2}{3}}}$

4. (8 points) Find all values of the constants m and b for which the function

$$y = \begin{cases} \sin x & x < \pi \\ mx + b & x \geq \pi \end{cases}$$

   will be differentiable at $x = \pi$.

5. (6 points) Find the linear approximation to $f(x) = \ln(1+x)$ at $x = 0$.
6. Find all of the functions having the following derivatives:
   (a) (4 points) $\frac{dy}{dx} = e^{2x}$
   (b) (5 points) $\frac{dy}{dx} = x^{-5} + \frac{1}{x}$
7. (5 points) If the acceleration is $a(t) = -4\sin 2t$, $v(0) = 2$, and $s(0) = -3$, find the position, $s(t)$ at any time t.
8. For the function f with $f'(x) = (x^2 + 2x - 3)e^{x}$, find the following:

(a) (5 points) critical points of $f(x)$

(b) (3 points) the interval(s) on which f is decreasing

(c) (2 points) the local maxima of f

Remember that $f'(x) = (x^2 + 2x - 3)e^x$. Find the following:

(d) (2 points) the points of inflection of f

(e) (5 points) the interval(s) on which f is concave up

Part III - Applications Show all work to receive credit. NO WORK= NO CREDIT.

9. (8 points) Two cars start moving from the same point. One travels south at 60 miles per hour and the other travels west at 25 miles per hour. At what rate is the distance between the cars increasing 3 hours later?

10. (9 points) Two sides of a triangle have lengths a and b, and the angle between them is θ. What value of θ will maximize the triangle's area? Recall that $A = \frac{1}{2}ab\sin\theta$.

Bonus (?? points) Use the Intermediate Value Theorem and Mean Value Theorem to show that

$$f(x) = x^3 + x - 1$$

has exactly one root between $x = 0$ and $x = 1$.