Math 142, Spring, 2006 **Review for Exam 2** Name:

Disclaimer See the disclaimer from first review sheet. The second exam will be on Thursday, 6 April, 2006. It will cover material up through section 4.5.

1. Be able to state the Extreme Value Theorem.
2. Be able to state Rolle's theorem.
3. Be able to state the Mean Value Theorem.
4. Be able to do word problems like the worksheet for related rate problems.
5. Be able to do word problems like the worksheet for optimization problems.
6. Find the values of θ, $\sin\theta$, and $\cos\theta$ given that:
   (a) $\theta = \sin^{-1}\frac{1}{2}$
   (b) $\theta = \sec^{-1}\sqrt{2}$
   (c) $\theta = \cot^{-1}\frac{1}{\sqrt{3}}$
7. Find $\frac{dy}{dx}$ for each of the following;
   (a) $xy + 2x + 3y = 1$
   (b) $\sqrt{xy} + y^2 = 1$
   (c) $\sin^2 x + \cos^2 y = 1$
   (d) $\frac{x^3 - xy}{x} = 3y$
   (e) $x(t) = 2t^7 - 4t^2 + 5$ $\quad y(t) = 3t^2 + 5$
   (f) $x(t) = \cos(5t^2 + t)$ $\quad y(t) = \cos^2(7t^3 - t^2)$
   (g) $y = x^3 - 3x^2 - 1$
8. Find equations of the tangent line to the curve at $x = a$.
   (a) $f(x) = -\left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{x^2}\right)$ for $a = 2$.
   (b) $g(x) = \cos\pi x + 3$ for $a = 2$.
9. Find the points where the tangent to the curve has the following properties:
   (a) $f(x) = -x^2 - 6$ is parallel to the line $y = 4x - 1$.
   (b) $g(x) = x^3 - 3x$ is perpendicular to the line $5y - 3x - 8 = 0$.
   (c) $g(x) = \dfrac{5x}{x^2 + 1}$ is horizontal.
10. Does $f(x) = \dfrac{x + 4}{x + 1}$ ever have a horizontal tangent line? Justify your answer.
11. (a) Find the intervals where $f(x) = \pi^{\cos x}$ is increasing and where it is decreasing.
    (b) Find the intervals where $f(x) = 2x^3 + 3x^2 - 37x + 24$ has slope less than or equal to -1.
12. Find all values of x where $f(x) = \dfrac{-5x}{5x - 15}$ is concave up.

13. A farmer wants to build a rectangular garden which is to have a 4 foot opening on one side. If the fence costs $3 per foot for the side with the opening and $2 per foot for the other 3 sides, find the dimensions of the largest garden he can build if he only has $300 to spend.

14. A closed rectangular box is to be constructed with a base that is twice as long as it is wide. If the total surface area is the be 24 square centimeters, find the dimensions of the box that will maximize the volume.

15. Find the most general antiderivative for the following functions:

    (a) $f(x) = 5x^{\frac{1}{4}} - 7x^{\frac{3}{4}}$
    (b) $f(t) = 7\cos t - 5\sin t$
    (c) $f(\theta) = e^{\theta} + \sec\theta\tan\theta$
    (d) $f(x) = \dfrac{x^2 + x + 1}{x}$

16. Find $f(x)$:

    (a) $f'(x) = \dfrac{4}{\sqrt{1 - x^2}}$ when $f(\frac{1}{2}) = 1$
    (b) $f''(x) = x^2 + 3\cos x$ when $f(1) = 1$ and $f'(1) = 5$

17. Differentiate $f(x) = \dfrac{x}{1 + x}$.

18. Differentiate $g(x) = \pi x^e - e^{\pi} x$.

19. Differentiate $h(x) = x \cdot \sin x$.

20. Differentiate $l(x) = \dfrac{\sin x}{1 + \cos x}$.

21. What can be said about the extreme values of a function that is continuous on a closed interval?

22. What is a differential equation? What is a solution to a differential equation?

23. If you set the second derivative equal to zero, solve the equation, and get the solution $x = c$, must $x = c$ be an inflection point? If yes, prove it. If no, give an example of a function $f(x)$ for which there is a c with $f''(c) = 0$ but c is not an inflection point.

24. Can different functions have the same derivative? If yes, give an example of two different functions with the same derivative. If no, prove that different functions cannot have the same derivative.

25. If $y = f(x)$ is a differentiable function, is it possible that there exists a point c such that $f'(c) < 0$ and $f''(c) > 0$? If yes, give an example. If no, explain why such a point cannot exist.

26. Find all possible functions with the given derivatives:

    (a) $f'(x) = \frac{1}{x^2} + \frac{1}{x} + 1 + x + x^2$
    (b) $f'(x) = \sec x \tan x$
    (c) $f'(x) = 5^x + xe^{-x^2}$
    (d) $f'(x) = \sin x$
    (e) $f'(x) = \sqrt{2x + 3} + \frac{1}{\sqrt{2x+3}}$