Math 142
Spring, 2004 Name: **Solutions**

Review for Exam 1 - Chapter 1, and Sections 2.1-2.6, and P.1, P.2, P.5, and P.6

Disclaimer The following are a sampling of problems that could appear on your first exam. The exam will be shorter than the list of questions which appear here. Some of these problems could appear on the first exam, and these highlight many of the topics which you should expect to see. This is by no means an exhaustive list, nor does it provide the only possible questions that you will see on the exam. You should also review examples from class notes, problems on quizzes, and (to a slightly lesser extent) problems from homework.

1. State the definition of a function.

   A function is a rule that assigns to each input a unique output.

2. State the Intermediate value theorem.

   Let $f(x)$ be continuous on $[a, b]$ and $f(a) < N < f(b)$. Then there exists $c \in (a, b)$ so that $f(c) = N$.

3. State the definition of a continuous function.

   A function is continuous at $x = a$ if $\lim_{x \to a} f(x) = f(a)$. A function is continuous if it is continuous for all $a \in \mathbb{R}$.

4. Know the unit circle.

   By this I mean, know the radian measures of the special angles, and be able to fill in the coordinates of the points at each of the special angles. Remember that $\cos\theta$ is the x coordinate of the point on the unit circle for the angle θ and $\sin\theta$ is the y coordinate.

5. State the formula for the slope of the tangent line to a function $f(x)$ at a point x.

   The slope of the tangent line (which also happens to be the derivative) is

   $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

   if the limit exists.

6. Know the definition of the derivative of a function at a point, and the derivative of a function.

   The derivative of $f(x)$ at $x = a$ is

   $$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

   if the limit exists. The derivative (function) of the function $f(x)$ is

   $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

   if the limit exists.

7. Compute the following limits:

(a) $\lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$

$$\begin{aligned}
\lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} &= \lim_{h\to 0} \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} \\
&= \lim_{h\to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h} \\
&= \lim_{h\to 0} \frac{\frac{x-x-h}{x(x+h)}}{h} \\
&= \lim_{h\to 0} \frac{-h}{x(x+h)} \cdot \frac{1}{h} \\
&= \lim_{h\to 0} \frac{-1}{x(x+h)} \\
&= -\frac{1}{x \cdot x} = \frac{-1}{x^2}
\end{aligned}$$

(b) $\lim_{h\to 0} \frac{\sqrt{h^2+4h+5} - \sqrt{5}}{h}$

$$\begin{aligned}
&= \lim_{h\to 0} \frac{\sqrt{h^2+4h+5} - \sqrt{5}}{h} \left(\frac{\sqrt{h^2+4h+5} + \sqrt{5}}{\sqrt{h^2+4h+5} + \sqrt{5}} \right) \\
&= \lim_{h\to 0} \frac{h^2+4h+5-5}{h\left(\sqrt{h^2+4h+5} + \sqrt{5}\right)} \\
&= \lim_{h\to 0} \frac{h^2+4h}{h\left(\sqrt{h^2+4h+5} + \sqrt{5}\right)} \\
&= \lim_{h\to 0} \frac{h+4}{\sqrt{h^2+4h+5} + \sqrt{5}} \\
&= \frac{4}{\sqrt{5} + \sqrt{5}} \\
&= \frac{4}{2\sqrt{5}} \\
&= \frac{2}{\sqrt{5}}
\end{aligned}$$

(c) $\lim_{x\to 2} \frac{|\, x-2 \,|}{x-2}$

$$\begin{aligned}
\lim_{x\to 2^+} \frac{|\, x-2 \,|}{x-2} &= \lim_{x\to 2^+} \frac{x-2}{x-2} \\
&= \lim_{x\to 2^+} 1 = 1
\end{aligned}$$

$$\lim_{x\to 2^-} \frac{|\, x-2 \,|}{x-2} = \lim_{x\to 2^-} \frac{-(x-2)}{x-2}$$
$$= \lim_{x\to 2^+} -1 = -1$$

Therefore the limit does not exist (since the two one-sided limits do not match)

(d) $\lim_{x\to 3} \sqrt{\frac{x+2}{x+1}}$

$$\lim_{x\to 3} \sqrt{\frac{x+2}{x+1}} = \sqrt{\lim_{x\to 3} \frac{x+2}{x+1}}$$

Since this is a rational function which is defined at $x = 3$ we can just plug in $x = 3$:

$$= \sqrt{\frac{3+2}{3+1}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$

(e) $\lim_{x\to 0^+} \sqrt{x} \sin\left(\frac{1}{x}\right)$

Use the sandwich theorem:

$$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$
$$-\sqrt{x} \leq \sqrt{x} \sin\left(\frac{1}{x}\right) \leq \sqrt{x}$$

As $x \to 0^+$ of each part of this gives;

$$0 \leq \lim_{x\to 0} \sqrt{x} \sin\left(\frac{1}{x}\right) \leq 0$$

Therefore by the sandwich theorem, we must have that

$$\lim_{x\to 0} \sqrt{x} \sin\left(\frac{1}{x}\right) = 0$$

(f) $\lim_{x\to\infty} \frac{x^7-1}{x^6+1}$

$$\lim_{x\to\infty} \frac{x^7-1}{x^6+1} = \lim_{x\to\infty} \frac{\frac{1}{x^6}\left(x^7-1\right)}{\frac{1}{x^6}\left(x^6-1\right)}$$
$$= \lim_{x\to\infty} \frac{x - \frac{1}{x^6}}{1 + \frac{1}{x^6}}$$
$$= \infty$$

(g) $\lim_{x\to\infty} \frac{x-1}{\sqrt{x^2+29}}$

$$= \lim_{x\to\infty} \frac{\frac{1}{x}(x-1)}{\frac{1}{x}\sqrt{x^2+29}}$$

$$= \lim_{x\to\infty} \frac{1-\frac{1}{x}}{\frac{1}{\sqrt{x^2}}\sqrt{x^2+29}}$$

$$= \lim_{x\to\infty} \frac{1-\frac{1}{x}}{\sqrt{1-\frac{29}{x^2}}}$$

$$= \frac{1}{\sqrt{1}}$$

$$= 1$$

(h) $\lim_{x\to\infty} \dfrac{3x^{29}+7}{3x^{29}+5}$

$$= \lim_{x\to\infty} \frac{\frac{1}{x^{29}}\left(3x^{29}+7\right)}{\frac{1}{x^{29}}\left(3x^{29}+5\right)}$$

$$= \lim_{x\to\infty} \frac{3+\frac{7}{x^{29}}}{3+\frac{5}{x^{29}}} = \frac{3}{3} = 1$$

8. Which of the above functions (in (a) thru (e)) are continuous at the points where you evaluated the limits?

   Notice that no function can be continuous at infinity because we can't evaluate the function at that point.

   (a) This is not continuous since $h = 0$ is not in the domain of the function.
   (b) This is not continuous since $h = 0$ is not in the domain of the function.
   (c) The limits don't match (and the point is not in the domain)
   (d) This is continuous
   (e) This is not continuous since $\frac{1}{x}$ is not defined at $x = 0$

9. Find the average rate of change of the function $f(x) = \cot t$ over the interval $[\frac{\pi}{4}, \frac{\pi}{2}]$.

$$f\left(\frac{\pi}{4}\right) = \cot\frac{\pi}{4} = \frac{\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

$$f\left(\frac{\pi}{2}\right) = \frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}} = \frac{0}{1} = 0$$

   so find the slope from the points $\left(\frac{\pi}{4}, 1\right)$ and $\left(\frac{\pi}{2}, 0\right)$:

$$\text{average rate of change: } \frac{f(\frac{\pi}{4}) - f(\frac{\pi}{2})}{\frac{\pi}{4} - \frac{\pi}{2}} = \frac{1-0}{\frac{\pi}{4}} = -\frac{4}{\pi}$$

10. Find the average rate of change of the function $f(t) = 16t^2$ over the interval $[2, 3]$.

$$f(2) = 16(2^2) = 64$$
$$f(3) = 16(3^2) = 16 \cdot 9 = 144$$
$$\text{a.r.c.} = \frac{f(3) - f(2)}{3 - 2} = \frac{144 - 64}{1} = 80$$

11. Find the instantaneous rate of change of the function $f(t) = 16t^2$ at $t = 3$.

This is the derivative of $f(t)$ at $t = 3$:

$$f'(t) = 16(2t) = 32t$$
$$f'(3) = 16(6) = 96$$

12. Write the equation of the tangent line to $f(t) = 16t^2$ at $t = 3$.

The slope is 96, and the line goes through the point $(3, 144)$, so the equation of the line is:

$$y - 144 = 96(x - 3)$$

13. Write the equation of the line perpendicular to the tangent line of $f(t) = 16t^2$ at $t = 3$.

This has slope $m = -\frac{1}{96}$, and goes through the point $(3, 144)$.

$$y - 144 = -\frac{1}{96}(x - 3)$$

This is also called the normal line to the curve at that point.

14. Find the values of θ, $\sin\theta$, and $\cos\theta$ given that:

(a) $\theta = \sin^{-1}\frac{1}{2}$

$$\theta = \frac{\pi}{6}$$
$$\sin\theta = \frac{1}{2}$$
$$\cos\theta = \frac{\sqrt{3}}{2}$$

(b) $\theta = \sec^{-1}\sqrt{2}$

$$\theta = \sec^{-1}\sqrt{2}$$
$$\sec\theta = \sqrt{2}$$
$$\frac{1}{\cos\theta} = \sqrt{2}$$
$$\cos\theta = \frac{\sqrt{2}}{2}$$
$$\theta = \frac{\pi}{4}$$
$$\sin\theta = \frac{\sqrt{2}}{2}$$

(c) $\theta = \cot^{-1} \frac{1}{\sqrt{3}}$

$$\begin{aligned}
\theta &= \cot^{-1} \frac{1}{\sqrt{3}} \\
\cot\theta &= \frac{1}{\sqrt{3}} \\
\frac{x}{y} = \frac{\cos\theta}{\sin\theta} &= \frac{1}{\sqrt{3}} \\
\theta &= \frac{\pi}{3} \\
\sin\theta &= \frac{\sqrt{3}}{2} \\
\cos\theta &= \frac{1}{2}
\end{aligned}$$

15. What is the relationship between the slope of the tangent line and the derivative?

    The slope of the tangent line at the point is equal to the derivative of the function at the point.

16. What is the relationship between the slope of the tangent line and the slope of the normal to the curve at a point $x = a$?

    The slope of the normal line is the negative reciprocal of the slope of the tangent line, so it is the negative reciprocal of the derivative of the function at that point.

17. A 45-caliber bullet fired straight up from the surface of the moon would reach a height of $s_1(t) = 832t - 2.6t^2$ after t seconds. On the earth (in a vacuum), the 45-caliber bullet fired straight up would reach a height of $s_2(t) = 832t - 16t^2$.

    (a) How fast is the bullet traveling when it reaches the highest point?
    This occurs when the velocity is 0.

    (b) What is the highest height reached by the bullet on the earth? On the moon?

On the moon:	On the earth:
$v(t) = 832 - 5.2t$	$v(t) = 832 - 32t$
$0 = 832 - 5.2t$	$0 = 832 - 32t$
$5.2t = 832$	$32t = 832$
$t \approx 160.58$	$t = 26$
$s_1(160.58) = 832(160.58) - 2.6(160.58)^2$	$s_2(26) = 832(26) - 16(26)^2$
≈ 66559.12 ft.	$= 10816$ ft

    (c) How long is the bullet aloft on the earth? On the moon?
    This asks for when the height is 0.

On the moon:	On the earth:
$0 = 832t - 2.6t^2$	$0 = 832t - 16t^2$
$0 = t(832 - 2.6t)$	$0 = t(832 - 16t)$
$t = 0$ or $t = 320$	$t = 0$ or $t = 52$

Therefore it is aloft for 320 seconds on the moon and 52 seconds on the earth.

18. The position of a particle is given by $s(t) = t^3 - 4.5t^2 - 7t$ at t seconds for $t \geq 0$. At what time is the particle's velocity 5 m/s?

$$v(t) = 3t^2 - 9t - 7$$
$$5 = 3t^2 - 9t - 7$$
$$3t^2 - 9t - 12 = 0$$
$$3(t^2 - 3t - 4) = 0$$
$$t^2 - 3t - 4 = 0$$
$$(t-4)(t+1) = 0$$
$$t = 4 \text{ or } t = -1$$

Therefore at time 4 seconds the particle's velocity is 5 m/s.

19. If a ball is thrown upward with a velocity of 80 feet per second, it's height t seconds after it is released is $s(t) = 80t - 16t^2$.

(a) What is the maximum height of the ball?

This happens when the velocity is 0:

$$v(t) = 80 - 32t$$
$$0 = 80 - 32t$$
$$80 = 32t$$
$$t = 2.5$$

$$s(2.5) = 80(2.5) - 16(2.5)^2 = 100$$

The maximum height of the ball is 100 ft.

(b) What is the velocity when the ball is at a height of 96 feet on the way up?

$$s(t) = 96$$
$$80t - 16t^2 = 96$$
$$16t^2 - 80t + 96 = 0$$
$$16(t^2 - 5t + 6) = 0$$
$$t^2 - 5t + 6 = 0$$
$$(t-2)(t-3) = 0$$
$$t = 2 \text{ or } t = 3$$

On the way up indicates we are interested in time $t = 2$:

$$v(2) = 80 - 32(2) = 16 \text{ ft/sec}$$

20. Compute the following derivatives:

(a) $\frac{d}{dx}(\frac{x^3}{2})$

$$\frac{d}{dx}\left(\frac{x^3}{2}\right) = \frac{1}{2}\left(3x^2\right) = x^2$$

(b) $\frac{d}{dx}\left(\frac{3x+2}{2x-11}\right)$

Use quotient rule:

$$\begin{aligned}\frac{d}{dx}\left(\frac{3x+2}{2x-11}\right) &= \frac{3(2x-11)-2(3x+2)}{(2x-11)^2} \\ &= \frac{6x-33-6x-4}{(2x-11)^2} \\ &= \frac{-37}{(2x-11)^2}\end{aligned}$$

(c) $\frac{d}{dx}(\sqrt{\sin^2 x})$

$$\frac{d}{dx}\left(\sqrt{\sin^2 x}\right) = \frac{d}{dx}(\sin x) = \cos x$$

(d) $\frac{d}{dx}\left((\sin^2(2x)+1)^e\right)$

Use chain rule

$$\begin{aligned}&= e\left(\sin^2(2x)+1\right)^{e-1} 2\sin(2x)\cdot\cos(2x)\cdot 2 \\ &= 4e\left(\sin^2(2x)+1\right)^{e-1}\sin(2x)\cos(2x)\end{aligned}$$

(e) $\frac{d}{dx}(\tan(\cos(\sin x)))$

Use chain rule

$$\begin{aligned}&= \sec^2(\cos(\sin x))\cdot -\sin(\sin x)\cdot\cos x \\ &= -\sec^2(\cos(\sin x))\sin(\sin x)\cos x\end{aligned}$$

(f) $\frac{d}{dx}((\tan x)(\cos x)(\sin x))$

This is product rule (twice)

$$= \sec^2 x\cos x\sin x - \tan x\sin^2 x + \tan x\cos^2 x$$

(g) $\frac{d}{dx}\left((4x+3)^4(x+1)^{-1}\right)$

$$16(4x+3)^3(x+1)^{-1} - (x+1)^{-2}(4x+3)^4$$

(h) $\frac{d}{dx}\left(\sin\left(\frac{3\pi x}{x}\right)+\cos\left(\frac{3\pi x}{2}\right)\right)$

$$\begin{aligned}&= \frac{d}{dx}\left(\sin(3\pi)+\cos\left(\frac{3\pi x}{2}\right)\right) \\ &= 0 - \frac{3\pi}{2}\sin\left(\frac{3\pi x}{2}\right)\end{aligned}$$

since the first is a constants

$$= -\frac{3\pi}{2}\sin\left(\frac{3\pi x}{2}\right)$$

(i) $\frac{d}{dx}\left(x\tan x\right)$
Use product rule:

$$= \tan x + x\sec^2 x$$

(j) $\frac{d}{dx}\left((x^2+1)\sec x\right)$
Use the product rule

$$= 2x\sec x + (x^2+1)\sec x\tan x$$

(k) $\frac{d}{dx}\left(x(x^2+1)\tan x\sec x\right)$

$$= \frac{d}{dx}\left((x^3+x)\tan x\sec x\right)$$

Use the product rule (three-product formula):

$$= (3x^2+1)\tan x\sec x + (x^3+x)\sec^2 x\sec x + (x^3+x)\tan x\sec x\tan x$$
$$= (3x^2+1)\tan x\sec x + (x^3+x)\sec^3 x + (x^3+x)\tan^2 x\sec x$$

21. Write the parametric equation for the line segment which starts at $(5,-2)$ and ends at $(3,4)$, and takes 3 seconds to walk.

$$x(t) = at + b \qquad\qquad y(t) = ct + d$$

At $t = 0, x = 5$ and $y = -2$

$$x(0) = a(0) + b = 5 \Rightarrow b = 5$$
$$y(0) = c(0) + d = -2 \Rightarrow d = -2$$
$$x(t) = at + 5$$
$$y(t) = ct - 2$$

At $t = 3, x = 3$ and $y = 4$

$$3 = x(3) = 3a + 5 \Rightarrow -2 = 3a \Rightarrow a = -\frac{2}{3}$$
$$4 = y(3) = 3c - 2 \Rightarrow 6 = 3c \Rightarrow c = 2$$

So we have

$$x(t) = -\frac{2}{3}t + 5 \qquad y(t) = 2t - 2 \qquad 0 \le t \le 3$$

22. Find the derivatives of all orders of each of these functions:

(a) $f(x) = \frac{1}{x}$

$$f'(x) = -x^{-2} = -\frac{1}{x^2}$$
$$f''(x) = 2x^{-3} = \frac{2}{x^3}$$
$$f'''(x) = -6x^{-4} = -\frac{6}{x^4}$$
$$f^{(n)}(x) = (-1)^n\frac{1\cdot 2\cdot 3\cdots n}{x^{n-1}}$$

(b) $f(x) = \sin x + \cos x$

$$\begin{aligned} f'(x) &= \cos x - \sin x \\ f''(x) &= -\sin x - \cos x \\ f'''(x) &= -\cos x + \sin x \\ f^{(4)}(x) &= \sin x + \cos x \end{aligned}$$

$$\begin{aligned} f^{(4n+1)} &= \cos x - \sin x \\ f^{(4n+2)} &= -\sin x - \cos x \\ f^{(n+3)} &= -\cos x + \sin x \\ f^{(4n)} &= \sin x + \cos x \end{aligned}$$

(c) $f(x) = \sin(2x)$

$$\begin{aligned} f'(x) &= 2\cos(2x) \\ f''(x) &= -4\sin(2x) \\ f'''(x) &= -8\cos(2x) \\ f^{(4)}(x) &= 16\sin(2x) \end{aligned}$$

I'll let you write out the pattern.

23. Use the definition of derivative to find $f'(x)$ for each of the following functions:

(a) $f(x) = \sqrt{x+2}$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h} \left(\frac{\sqrt{x+h+2} + \sqrt{x+2}}{\sqrt{x+h+2} + \sqrt{x+2}} \right) \\ &= \lim_{h\to 0} \frac{x+h+2-x-2}{h(\sqrt{x+h+2} + \sqrt{x+2})} \\ &= \lim_{h\to 0} \frac{-h}{h(\sqrt{x+h+2} + \sqrt{x+2})} \\ &= \lim_{h\to 0} \frac{-1}{\sqrt{x+h+2} + \sqrt{x+2}} \\ &= \frac{-1}{\sqrt{x+2} + \sqrt{x+2}} \\ &= \frac{-1}{2\sqrt{x+2}} \end{aligned}$$

(b) $f(x) = \frac{1}{x}$

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\ &= \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} \\ &= \frac{\frac{x-x-h}{x(x+h)}}{h} \\ &= \lim_{h \to 0} \frac{-h}{x(x+h)} \cdot \frac{1}{h} \\ &= \lim_{h \to 0} \frac{-1}{x(x+h)} \\ &= \frac{-1}{x^2} \end{aligned}$$

(c) $f(x) = mx + b$, where m and b are constants.

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{m(x+h) + b - (mx + b)}{h} \\ &= \lim_{h \to 0} \frac{mx + mh + b - mx - b}{h} \\ &= \lim_{h \to 0} \frac{mh}{h} \\ &= \lim_{h \to 0} m \\ &= m \end{aligned}$$

(d) $f(x) = ax^2 + bx + c$ where a, b, and c are constants.

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{a(x+h)^2 + b(x+h) + c - ax^2 - bx - c}{h} \\ &= \lim_{h \to 0} \frac{ax^2 + 2axh + 2h^2 + bx + bh + c - ax^2 - bx - c}{h} \\ &= \lim_{h \to 0} \frac{2axh + 2h^2 + bh}{h} \\ &= \lim_{h \to 0} 2ax + 2h + b \\ &= 2ax + b \end{aligned}$$

(e) $f(x) = x^3$

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} \\ &= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \end{aligned}$$

$$= \lim_{h \to 0} \frac{3x^2h + 2xh^2 + h^3}{h}$$

$$= \lim_{h \to 0} 3x^2 + 2xh + h^2$$

$$= 3x^2$$

(f) $f(x) = \frac{1}{x-3}$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h-3} - \frac{1}{x-3}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x-3-x-h-3}{(x-3)(x+h-3)}}{h}$$

$$= \lim_{h \to 0} \frac{-h}{(x-3)(x+h-3)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-1}{(x-3)(x+h-3)}$$

$$= \frac{-1}{(x-3)^2}$$

24. Use the shortcuts that we know to check your answers above.

25. Find $f'(x)$ and $f''(x)$ for each of these functions:

(a) $f(x) = 7x - 3$

$$f'(x) = 7$$

$$f''(x) = 0$$

(b) $f(x) = e^\pi$

$$f'(x) = 0$$

$$f''(x) = 0$$

(c) $f(x) = \frac{1}{\sqrt{x}}$

$$f'(x) = -\frac{1}{x^2} = -x^{-2}$$

$$f''(x) = 2x^{-3} = \frac{2}{x^3}$$

(d) $f(x) = 5\sqrt[3]{x} + 6x^2 - e$

$$f'(x) = \frac{5}{3}x^{-\frac{2}{3}} + 12x$$

$$f''(x) = -\frac{5}{9}x^{-\frac{5}{3}} + 12$$

(e) $f(x) = \frac{1}{x^2} + \frac{1}{x} + 1$

$$f(x) = x^{-2} + x^{-1} + 1$$
$$f'(x) = -2x^{-3} - x^{-2} = -\frac{2}{x^3} - \frac{1}{x^2}$$
$$f''(x) = 6x^{-4} + 2x^{-3} = \frac{6}{x^4} + \frac{2}{x^3}$$

(f) $f(x) = \sin 3$

$$f'(x) = 0$$
$$f''(x) = 0$$

(g) $f(x) = x^2 \sin 2$

$$f'(x) = \sin 2 \cdot 2x = 2x \sin 2$$
$$f''(x) = 2 \sin 2$$

(h) $f(x) = x^e + x^\pi$

$$f'(x) = ex^{e-1} + \pi x^{\pi-1}$$
$$f''(x) = e(e-1)x^{e-2} + \pi(\pi-1)x^{\pi-2}$$

26. Find the following derivatives:

(a) $\frac{d}{dt}\left(\frac{t^2+3t}{t}\right)$

$$= \frac{d}{dt}(t+3)$$
$$= 1$$