Math 142
Spring, 2005 Name:

Review for Exam 1 - Chapter 1, and Sections 2.1-2.5, and P.1, P.2, P.5, and P.6

Disclaimer The following are a sampling of problems that could appear on your first exam. The exam will be shorter than the list of questions which appear here. Some of these problems could appear on the first exam, and these highlight many of the topics which you should expect to see. This is by no means an exhaustive list, nor does it provide the only possible questions that you will see on the exam. You should also review examples from class notes, problems on quizzes, and (to a slightly lesser extent) problems from homework.

1. State the definition of a function.
2. State the Intermediate value theorem.
3. State the definition of a continuous function.
4. Know the unit circle.
5. State the formula for the slope of the tangent line to a function $f(x)$ at a point x.
6. Know the definition of the derivative of a function at a point, and the derivative of a function.
7. Be able to solve word problems like in section 2.2. using velocity, acceleration, and jerk.
8. Compute the following limits:
   (a) $\lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$
   (b) $\lim_{h \to 0} \frac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$
   (c) $\lim_{x \to 2} \frac{| x - 2 |}{x - 2}$
   (d) $\lim_{x \to 3} \sqrt{\frac{x+2}{x+1}}$
   (e) $\lim_{x \to 0} \sqrt{x} \sin\left(\frac{1}{x}\right)$
   (f) $\lim_{x \to \infty} \frac{x^7 - 1}{x^6 + 1}$
   (g) $\lim_{x \to \infty} \frac{x - 1}{\sqrt{x^2 + 29}}$
   (h) $\lim_{x \to \infty} \frac{3x^{29} + 7}{3x^{29} + 5}$
9. Which of the above functions (in (a) thru (e)) are continuous at the points where you evaluated the limits?

10. Find the average rate of change of the function $f(x) = \cot t$ over the interval $[\frac{\pi}{4}, \frac{\pi}{2}]$.

11. Find the average rate of change of the function $f(t) = 16t^2$ over the interval $[2, 3]$.

12. Find the instantaneous rate of change of the function $f(t) = 16t^2$ at $t = 3$.

13. Write the equation of the tangent line to $f(t) = 16t^2$ at $t = 3$.

14. Write the equation of the line perpendicular to the tangent line of $f(t) = 16t^2$ at $t = 3$.

15. Find the values of θ, $\sin\theta$, and $\cos\theta$ given that:

    (a) $\theta = \sin^{-1}\frac{1}{2}$

    (b) $\theta = \sec^{-1}\sqrt{2}$

    (c) $\theta = \cot^{-1}\frac{1}{\sqrt{3}}$

16. What is the relationship between the slope of the tangent line and the derivative?

17. What is the relationship between the slope of the tangent line and the slope of the normal to the curve at a point $x = a$?

18. A 45-caliber bullet fired straight up from the surface of the moon would reach a height of $s_1(t) = 832t - 2.6t^2$ after t seconds. On the earth (in a vacuum), the 45-caliber bullet fired straight up would reach a height of $s_2(t) = 832t - 16t^2$.

    (a) How fast is the bullet traveling when it reaches the highest point?

    (b) What is the highest height reached by the bullet on the earth? On the moon?

    (c) How long is the bullet aloft on the earth? On the moon?

19. The position of a particle is given by $s(t) = t^3 - 4.5t^2 - 7t$ at t seconds for $t \geq 0$. At what time is the particle's velocity 5 m/s?

20. If a ball is thrown upward with a velocity of 80 feet per second, it's height t seconds after it is released is $s(t) = 80t - 16t^2$.

    (a) What is the maximum height of the ball?

    (b) What is the velocity when the ball is at a height of 96 feet on the way up?

21. Compute the following derivatives:

    (a) $\frac{d}{dx}(\frac{x^3}{2})$

    (b) $\frac{d}{dx}\left(\frac{3x+2}{2x-11}\right)$

    (c) $\frac{d}{dx}(\sqrt{\sin^2 x})$

    (d) $\frac{d}{dx}\left((\sin^2(2x)+1)^e\right)$

    (e) $\frac{d}{dx}(\tan(\cos(\sin x)))$

    (f) $\frac{d}{dx}((\tan x)(\cos x)(\sin x))$

(g) $\frac{d}{dx}\left((4x+3)^4(x+1)^{-1}\right)$

(h) $\frac{d}{dx}\left(\sin\left(\frac{3\pi x}{x}\right)+\cos\left(\frac{3\pi x}{2}\right)\right)$

(i) $\frac{d}{dx}\left(x\tan x\right)$

(j) $\frac{d}{dx}\left((x^2+1)\sec x\right)$

(k) $\frac{d}{dx}\left(x(x^2+1)\tan x\sec x\right)$

22. Write the parametric equation for the line segment which starts at $(5,-2)$ and ends at $(3,4)$, and takes 3 seconds to walk.

23. Find the derivatives of all orders of each of these functions:

(a) $f(x)=\frac{1}{x}$

(b) $f(x)=\sin x+\cos x$

(c) $f(x)=\sin(2x)$

24. Use the definition of derivative to find $f'(x)$ for each of the following functions:

(a) $f(x)=\sqrt{x+2}$

(b) $f(x)=\frac{1}{x}$

(c) $f(x)=mx+b$, where m and b are constants.

(d) $f(x)=ax^2+bx+c$ where a, b, and c are constants.

(e) $f(x)=x^3$

(f) $f(x)=\frac{1}{x-3}$

25. Use the shortcuts that we know to check your answers above.

26. Find $f'(x)$ and $f''(x)$ for each of these functions:

(a) $f(x)=7x-3$

(b) $f(x)=e^{\pi}$

(c) $f(x)=\frac{1}{\sqrt{x}}$

(d) $f(x)=5\sqrt[3]{x}+6x^2-e$

(e) $f(x)=\frac{1}{x^2}+\frac{1}{x}+1$

(f) $f(x)=\sin 3$

(g) $f(x)=x^2\sin 2$

(h) $f(x)=x^e+x^{\pi}$

27. Find $\frac{d}{dt}\left(\frac{t^2+3t}{t}\right)$.