Math 142
Spring, 2006 Name: **Solutions**
Related Rates Worksheet

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1. A baseball diamond is a square with side 90 feet. A batter hits the ball and runs toward first base with a speed of 24 feet per second. At what rate is his distance from second base decreasing when he is halfway to first base?

   Let x be the distance from 1st base and c be the distance from 2nd base. Then there is an equation:

$$x^2 + 90^2 = c^2$$

   when the runner is halfway to 1st, this is:

$$45^2 + 90^2 = c^2$$

$$10125 = c^2$$

$$c \approx 100.623$$

   To find $\frac{dc}{dt}$, take derivatives of the above equation:

$$2x\frac{dx}{dt} + 0 = 2c\frac{dc}{dt}$$

$$(2)(45)(24) = 2(100.623)\frac{dc}{dt}$$

$$\frac{dc}{dt} \approx 10.73 \text{ ft / sec}$$

2. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 kilometers per hour, and ship B is sailing north at 25 kilometers per hour. How fast is the distance between the ships changing at 4:00 pm that day?

   Let a be the distance travelled by ship A and b be the distance travelled by ship B. Let the distance between them be c. Then you have:

$$(a + b)^2 + 100^2 = c^2$$

   Using the fact that after 4 hours $a = 100$ and $b = 140$. Therefore,

$$240^2 + 100^2 = c^2$$

$$c^2 = 67600$$

$$c = 260$$

   Taking the derivative of the original equation, we have:

$$2(a + b)\left(\frac{da}{dt} + \frac{db}{dt}\right) + 0 = 2c\frac{dc}{dt}$$

$$2(240)(25+35) = 2(260)\frac{dc}{dt}$$

$$28800 = 520\frac{dc}{dt}$$

$$\frac{dc}{dt} \approx 55.3846 \text{ km/hr}$$

3. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 feet per second, how fast is the angle between the top of the ladder and the wall changing when the angle is $\frac{\pi}{4}$ radians?

   Let θ be the angle between the top of the ladder and the wall, and x the distance from the ladder to the wall. Then we have:

$$\sin\theta = \frac{x}{10}$$

   Taking derivatives:

$$\cos\theta\frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}$$

   Plugging in what we know:

$$\cos\frac{\pi}{4}\frac{d\theta}{dt} = \frac{1}{10}(2)$$

$$\frac{\sqrt{2}}{2}\frac{d\theta}{dt} = \frac{1}{5}$$

$$\frac{d\theta}{dt} = \frac{2}{5\sqrt{2}} \approx .2828 \text{ rad/sec}$$

4. The coordinates of a particle in the metric $xy-$plane are differentiable functions of time t with $\frac{dx}{dt} = -1$ m/sec and $\frac{dy}{dt} = -5$ m/sec. How fast is the particle's distance from the origin changing as it passes through the point $(5, 12)$?

   The distance between the particle and the origin is given by

$$D = \sqrt{x^2+y^2}$$

   Taking derivatives we have:

$$\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{dD}{dt} = 2(5)(-1) + 2(12)(-5)$$

$$\frac{dD}{dt} = -10 - 130 = -140 \text{ m/sec}$$