Math 142
Spring, 2004
Exam 2
Chapters 2 and 3

Part I - Definitions and Theorems State the requested definition or theorem. (3 points each)

1. (3 points) State the Mean Value Theorem.
2. (3 points) State the Extreme Value Theorem.

Part II - Calculations -Show all work to receive credit. NO WORK = NO CREDIT.

3. Compute the following derivatives:
   (a) (7 points) $\frac{d}{dx}\left(x^{\pi} + \sin\pi + \cos^{-1}\pi + \ln\pi + \ln(\pi x) + e^x + \pi^x\right)$
   (b) (8 points) $\frac{d}{dx}\left(\tan^{-1}\sqrt{x^2-1} + \sin^{-1}x\right)$
   (c) (4 points) $\frac{d}{dx}\left(\ln 10^x\right)$
   (d) (5 points) $f(x) = (e^x + \arccos x)\left(\frac{3}{x^2}\right)$. Find $f'(x)$.
   (e) (8 points) $\frac{d}{dx}\left(\sqrt[3]{\frac{(3-x)^4(x^2+1)}{(2x+5)^3}}\right)$
4. (8 points) Find all points where the tangent to the curve $f(x) = x^x$ is parallel to the line $y - 2x = 12$.
5. (6 points) Find the linear approximation to $f(x) = \frac{1}{\sqrt{1+x}}$ at $x = 1$.
6. Find all of the functions having the following derivatives:
   (a) (4 points) $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$
   (b) (5 points) $\frac{dy}{dx} = \sec(2\pi x)\tan(2\pi x)$ and $y(0) = 30$
   (c) (5 points) $\frac{dy}{dx} = \frac{1}{x}$ and $y(1) = 4$
7. For the function $f(x) = x\sin x$ and $0 \le x \le 2\pi$, find the following:
   (a) (5 points) critical points of f
   (b) (3 points) the interval(s) on which f is decreasing
   (c) (2 points) the local maxima of f

   Remember that $f(x) = x\sin x$ and $0 \le x \le 2\pi$. Find the following:
   (d) (5 points) the interval(s) on which f is concave up
   (e) (2 points) the points of inflection of f

Part III - Applications Show all work to receive credit. NO WORK= NO CREDIT.

8. (8 points) At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 kilometers per hour, and ship B is sailing north at 25 kilometers per hour. How fast is the distance between the ships changing at 4:00 pm that day?

9. (9 points) An isosceles triangle (two sides are the same length) has its vertex at the origin and its base parallel to the $x-$axis on the curve $y = 27 - x^2$. Find the largest area the triangle can have.