Math 142
Spring, 2005 Name:
Optimization Problems

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1. Find the maximum slope of the graph $f(x) = 24x^2 - x^4$ for $x \geq 0$.

2. Find constants a and b so that $f(x) = a + bx - x^2$ has a local maximum at $(-1, 2)$.

3. Find the dimensions of the rectangle with area 100 cm whose perimeter is minimized.

4. A cylindrical bottle is to be designed so it holds 100 cubic centimeters of perfume. Find the dimensions of the bottle so the amount of material used in the sides and the bottom is as small as possible. (Hint: A cylinder of height h and radius r has volume $\pi r^2 h$, lateral surface area $2\pi rh$, and surface area of to or bottom πr^2.)

5. Suppose the bottle from the previous problem is to be a rectangular box with square base in shape. Find the dimensions of the bottle which minimizes the amount of material used in the sides and bottom. Which shape uses less material?

6. You are in charge of security for a concert. You have 43 security guards to be placed at 25 foot increments around 3 sides of the rectangular audience area. If the fire marshal declares that the density of the audience can be no more than 25 people per 100 square feet, what is the maximum number of people who can attend the concert?

7. The demand equation for a certain product is $p = 6 - \frac{1}{2}x$ where x is the number of units sold and p is the price. Find the level of production which results in maximum revenue. (Hint: revenue is price times quantity sold.)

8. Find the dimensions of a closed rectangular box with square base and volume 800 cubic centimeters which is constructed from the least amount of material.

9. A rancher will make 2 corrals from 215 meters of fence. One corral is a square. the other corral is a rectangle with length 1.5 times its width. What are the dimensions of each corral resulting in the greatest combined area?