Math 142, Spring, 2006 **Review for Final** Name:

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1. State the Intermediate value theorem.
2. State the definition of a continuous function.
3. Know the unit circle.
4. State the formula for the slope of the tangent line to a function $f(x)$ at a point x.
5. Know the definition of the derivative of a function at a point, and the derivative of a function.
6. Be able to state the Extreme Value Theorem.
7. Be able to state Rolle's theorem.
8. Be able to state the Mean Value Theorem.
9. Calculate the derivatives of the following functions:
   (a) $y = \sqrt{x^2 + e^x}$
   (b) $y = \ln x^{500}$
   (c) $\cos(xy) = \sin(xy)$
   (d) $\dfrac{y^2 + 5\tan x - \frac{x}{y}}{x^4} = x$
   (e) $24x^3 - 3x^{-7} = y$
   (f) $f(x) = (e^x + \arcsin x)\left(\dfrac{3}{x^2}\right)$
10. Find equations of the tangent line to the curve at $x = a$.
    (a) $f(x) = -\left(1 + \dfrac{1}{x}\right)\left(1 + \dfrac{1}{x^2}\right)$ for $a = 2$.
    (b) $g(x) = \cos \pi x + 3$ for $a = 2$.
11. Find the points where the tangent to the curve has the following properties:
    (a) $f(x) = -x^2 - 6$ is parallel to the line $y = 4x - 1$.
    (b) $g(x) = x^3 - 3x$ is perpendicular to the line $5y - 3x - 8 = 0$.
    (c) $g(x) = \dfrac{5x}{x^2 + 1}$ is horizontal.
12. Are the following functions continuous at $x = a$? Are they differentiable at $x = a$?
    (a) $f(x) = \begin{cases} x^2 & x \le 1 \\ 2 - x & x > 1 \end{cases}$ at $a = 1$
    (b) $g(x) = \begin{cases} x - 1 & x \le -1 \\ (x-1)^2 & x \ge -1 \end{cases}$ at $a = -1$
    (c) $h(x) = \begin{cases} 4x & x < 1 \\ 2x^2 + 2 & x \ge 1 \end{cases}$ at $a = 1$

13. Does $f(x) = \dfrac{x+4}{x+1}$ ever have a horizontal tangent line? Justify your answer.

14. (a) Find the intervals where $f(x) = \pi^{\cos x}$ is increasing and where it is decreasing.

    (b) Find the intervals where $f(x) = 2x^3 + 3x^2 - 37x + 24$ has slope less than or equal to -1.

15. Find all values of x where $f(x) = \dfrac{-5x}{5x-15}$ is concave up.

16. If $f(x)$ is increasing for all x, find where $f \circ f$ is increasing.

17. Let $f(x) = \sqrt{x}$ Use the tangent line to f at $x = 64$ to approximate $\sqrt{65}$.

18. A farmer wants to build a rectangular garden which is to have a 4 foot opening on one side. If the fence costs $3 per foot for the side with the opening and $2 per foot for the other 3 sides, find the dimensions of the largest garden he can build if he only has $300 to spend.

19. A closed rectangular box is to be constructed with a base that is twice as long as it is wide. If the total surface area is the be 24 square centimeters, find the dimensions of the box that will maximize the volume.

20. Use the definition of the derivative to prove that if a function is periodic, then its derivative is periodic with the same period.

21. Find the most general antiderivative for the following functions:

    (a) $f(x) = 5x^{\frac{1}{4}} - 7x^{\frac{3}{4}}$

    (b) $f(t) = 7\cos t - 5\sin t$

    (c) $f(\theta) = e^{\theta} + \sec\theta\tan\theta$

    (d) $f(x) = \dfrac{x^2+x+1}{x}$

22. Find $f(x)$:

    (a) $f'(x) = \dfrac{4}{\sqrt{1-x^2}}$ when $f(\frac{1}{2}) = 1$

    (b) $f''(x) = x^2 + 3\cos x$ when $f(1) = 1$ and $f'(1) = 5$

23. Give a PRECISE definition of $f(x)$ is continuous at a.

24. Give a PRECISE definition of $f'(a)$.

25. True/False: If a function is differentiable at a, it is continuous at a. Provide a counterexample if this is false, i.e. give a function $f(x)$ which is differentiable at $x = a$, but not continuous there.

26. True/False: If a function is continuous at a, it is differentiable at a. Provide a counterexample if this is false, i.e. give a function $f(x)$ which is continuous at $x = a$, but not differentiable there.

    Evaluate the following limits.

27. $\displaystyle\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$

28. $\lim_{t\to 1^-} \frac{|t-1|}{t-1}$

29. $\lim_{x\to -\infty} \frac{\sqrt[3]{x^2}}{x-5}$

30. $\lim_{m\to 0} \cos m$

31. $\lim_{p\to 2} \frac{p^2-5p+6}{p-2}$

32. $\lim_{w\to -2} \frac{|w|-w}{w}$

33. $\lim_{x\to \infty} \sqrt{\frac{2+3x}{1-5x}}$

34. $\lim_{h\to 0} \frac{(h-2)^2-4}{h}$

35. $\lim_{x\to -\infty} \frac{\sin x}{x}$

36. $\lim_{b\to 0} \frac{1-\sqrt{1-b^2}}{b}$

37. $\lim_{x\to 1^+} f(x)$ where $f(x) = \begin{cases} (x-1)^3 & x < -1 \\ (x+1)^3 & x \geq -1 \end{cases}$

38. $\lim_{x\to \infty} \frac{\sqrt{5-4x+2x^2}}{x-3}$

39. Differentiate $f(x) = \frac{x}{1+x}$.

40. Differentiate $g(x) = \pi x^e - e^\pi x$.

41. Differentiate $h(x) = x \cdot \sin x$.

42. Differentiate $l(x) = \frac{\sin x}{1+\cos x}$.

43. Be able to do the derivatives on the derivative review sheet.

44. At what value(s) of x does the curve $y = x^3 + 2x$ have a tangent line parallel to the line $y = x$?

45. For what value(s) of a will the function $f(x) = \begin{cases} e^x & x < 0 \\ a + x & x \geq 0 \end{cases}$ be continuous at $x = 0$?

46. A ball is tossed from a bridge 45 ft. high. The height (in feet) of the ball after t seconds is given by $s(t) = -16t^2 + 105t + 45$. What is the ball's velocity at $t = 4$?

47. Use the definition of $f'(a)$ to find $f'(x)$ when $f(x) = 2x^2 - 3x + 19$. Find the equation of the tangent line to $f(x)$ at $a = 0$

48. What is the difference between average rate of change and instantaneous rate of change?

49. Let $f(x)$ be a function. If $\lim_{x \to c} f(x) = L$, must it be true that $f(c) = L$? Justify your answer.

50. What can be said about the extreme values of a function that is continuous on a closed interval?

51. State the Mean Value theorem. State Rolle's Theorem.

52. If you set the second derivative equal to zero, solve the equation, and get the solution $x = c$, must $x = c$ be an inflection point? If yes, prove it. If no, give an example of a function $f(x)$ for which there is a c with $f''(c) = 0$ but c is not an inflection point.

53. Can different functions have the same derivative? If yes, give an example of two different functions with the same derivative. If no, prove that different functions cannot have the same derivative.

54. How is the derivative of a function at a particular value of x related to the slope of the tangent line at that point? how is the derivative of a function at a particular value of x related to the slope of the normal line at that point?

55. What is implicit differentiation? When do you need it?

56. What is logarithmic differentiation? When is it most often used?

57. What is the difference, if any, between $\sin^{-1} x$ and $\frac{1}{\sin x}$?

58. What is the difference, if any, between $2 \sin x$ and $\sin 2x$?

59. If $y = f(x)$ is a differentiable function, is it possible that there exists a point c such that $f'(c) < 0$ and $f''(c) > 0$? If yes, give an example. If no, explain why such a point cannot exist.

60. Is the function $f(x) = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$ continuous at $x = 0$? Is it differentiable at $x = 0$?

61. Find all possible functions with the given derivatives:

    (a) $f'(x) = \frac{1}{x^2} + \frac{1}{x} + 1 + x + x^2$
    (b) $f'(x) = \sec x \tan x$
    (c) $f'(x) = 5^x + xe^{-x^2}$
    (d) $f'(x) = \sin x$
    (e) $f'(x) = \sqrt{2x+3} + \dfrac{1}{\sqrt{2x+3}}$

62. Evaluate the following integrals. Use differentiation to check your answers:

    (a) $\displaystyle\int xe^{-x^2} dx$
    (b) $\displaystyle\int \cos x dx$
    (c) $\displaystyle\int \frac{3}{(2-x)^2} dx$
    (d) $\displaystyle\int \tan x dx$

(e) $\int \frac{1}{\sqrt{1-3x}}dx$

(f) $\int \frac{1}{\sqrt{1-9x^2}}dx$

(g) $\int (e^{-x} + 4^x)dx$

(h) $\int (\cos^2 x + \sin^2 x)dx$

(i) $\int \pi dx$

(j) $\int \frac{4}{x + x\ln^2 x}dx$

(k) $\int \frac{x^2+4}{x}dx$

63. Find the average value of $f(x)$ over the given interval where $f(x) = -x^2 + 10x + 11$ on the interval $[0, 10]$

64. Let $f(t) = \frac{1}{2}\sin^2 t$. Then $F'(t) = \sin t \cos t$. Use the Fundamental Theorem of Calculus to find $\int_{\frac{\pi}{2}}^{\pi} \sin t \cos t \; dt$.

65. If the average value of $f(x)$ over $[-3, 5]$ is 4, what is $\int_{-3}^{5} f(x)dx$?

66. Compute the following indefinite integrals:

(a) $\int \frac{\csc\theta}{\csc\theta - \sin\theta}d\theta$

(b) $\int \frac{e^x}{1+e^{2x}}dx$

(c) $\int x^{-2} + x^3 + 2x + 5 \; dx$

(d) $\int e^x + x^e + edx$

(e) $\int \frac{1}{x}dx$

(f) $\int 3\sin x - 5\cos x \; dx$

(g) $\int x(x^2-3)^{49} \; dx$

(h) $\int \frac{x-2}{x^2-4x} \; dx$

(i) $\int x\sin(x^2) \; dx$

67. Compute the following definite integrals:

(a) $\int_{-4}^{6} |\, x+2 \,| \; dx$

(b) $\int_{0}^{\frac{\pi}{2}} e^{\sin x} \cos x \; dx$

(c) $\int_{0}^{10} -x^2 + 10x + 11 \; dx$

(d) $\int_1^2 \sqrt{x}\ dx$

(e) $\int_2^6 3t^2 + 4t\ dt$

(f) $\int_0^3 x(x^2 - 3)^{49}\ dx$

(g) $\int_1^4 \frac{x-2}{x^2-4x}\ dx$

(h) $\int_{\frac{\pi}{2}}^{\pi} x\sin(x^2)\ dx$

68. Suppose that f and g are continuous functions and that $\int_0^2 f(x)dx = \sqrt{2}$, $\int_0^5 f(x)dx = \sqrt{5}$ and $\int_0^2 g(x)dx = 1$. Find the following:

(a) $\int_0^2 (7f(x) - 11g(x))dx$

(b) $\int_2^5 f(x)dx$

69. Compute the following derivatives:

(a) $\frac{d}{dx}\int_1^x (t^2-1)^{19}dt$

(b) $\frac{d}{dx}\int_0^x \sin\theta^2 d\theta$

(c) $\frac{d}{dx}\int_x^{\pi} \frac{1}{1+t^4}dt$

(d) $\frac{d}{dx}\int_1^{\sqrt{x}} \frac{r^2}{r^2+1}dr$

(e) $\frac{d}{dx}\int_{-5}^{\sin x} u\cos u^3 du$

(f) $\frac{d}{dx}\int_2^{129} \sin x dx$

70. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the $x-$axis and the line $y = x - 2$.

71. Find the area of the region bounded by the curves $y = x^3$ and $x = y^3$.