
Definition 1 Let f be a function defined on some open interval that contains a number a , except possibly a itself. Then $\lim_{x \rightarrow a} f(x) = L$ if for every number $\epsilon > 0$ there exists a corresponding number $\delta > 0$ so that if

$$0 < |x - a| < \delta$$

then

$$|f(x) - L| < \epsilon$$

Equivalently, if $-\delta < x - a < \delta$ then $-\epsilon < f(x) - L < \epsilon$.

Or, if $\delta < x - a < \delta$ then $L - \epsilon < f(x) < L + \epsilon$.

i.e., if we are given a small interval around L then we can find a small interval around a so that f maps $(a - \delta, a + \delta)$ into $(L - \epsilon, L + \epsilon)$. And this must work for every epsilon.

1. Look at the function $f(x) = \sqrt{x - 3}$. We know that the

$$\lim_{x \rightarrow 7} \sqrt{x - 3} = 2$$

- (a) How close does x need to be to 7 for $f(x)$ to be within .001 of 2?
- (b) How close does x need to be to 7 for $f(x)$ to be within ϵ of 2?

2. Look at the function $f(x) = -3x - 2$. We know that

$$\lim_{x \rightarrow -1} f(x) = 1$$

- (a) How close does x have to be to -1 for $f(x)$ to be within 0.03 of 1?
- (b) How close does x have to be to -1 for $f(x)$ to be within ϵ of 1?

3. Look at the function $f(x) = \frac{4}{x}$. We know

$$\lim_{x \rightarrow 2} \frac{4}{x} = 2$$

- (a) How close does x have to be to 2 for $f(x)$ to be within 0.4 of 2?
- (b) How close does x have to be to 2 for $f(x)$ to be within ϵ of 2?