

1. Calculate the derivatives of the following functions. Wherever necessary, assume that  $y$  is a function of  $x$ .

(a)  $y = \sqrt{x^2 + e^x}$

$$y' = \frac{1}{2} (x^2 + e^x)^{-\frac{1}{2}} (2x + e^x)$$

(b)  $y = \ln x^{500}$

$$y = 500 \ln x$$
$$y' = 500 \cdot \frac{1}{x} = \frac{500}{x}$$

(c)  $\cos(xy) = \sin(xy)$

$$-\sin(xy) \left( y + x \frac{dy}{dx} \right) = \cos(xy) \left( y + x \frac{dy}{dx} \right)$$
$$-y \sin(xy) - x \sin(xy) \frac{dy}{dx} = y \cos(xy) + x \cos(xy) \frac{dy}{dx}$$
$$-y \sin(xy) - y \cos(xy) = x \cos(xy) \frac{dy}{dx} + x \sin(xy) \frac{dy}{dx}$$
$$-y \sin(xy) - y \cos(xy) = (x \cos(xy) + x \sin(xy)) \frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{-y \sin(xy) - y \cos(xy)}{x \cos(xy) + x \sin(xy)}$$

(d)  $\frac{y^2 + 5 \tan x - \frac{x}{y}}{x^4} = x$

$$y^2 + 5 \tan x - \frac{x}{y} = x^5$$
$$2y \frac{dy}{dx} + 5 \sec^2 x - \frac{y - x \frac{dy}{dx}}{y^2} = 5x^4$$
$$2y \frac{dy}{dx} + 5 \sec^2 x - \frac{1}{y} + \frac{x}{y^2} \frac{dy}{dx} = 5x^4$$
$$\left( 2y + \frac{x}{y^2} \right) \frac{dy}{dx} = 5x^4 - 5 \sec^2 x + \frac{1}{y}$$
$$\frac{dy}{dx} = \frac{5x^4 - 5 \sec^2 x + \frac{1}{y}}{2y + \frac{x}{y^2}}$$

(e)  $24x^3 - 3x^{-7} = y$

$$y' = 72x^2 + 21x^{-8}$$

(f)  $f(x) = (e^x + \arcsin x)\left(\frac{3}{x^2}\right)$

$$f'(x) = \left(e^x + \frac{1}{\sqrt{1-x^2}}\right) \left(\frac{3}{x^2}\right) + (e^x + \sin^{-1} x) \left(\frac{-6}{x^3}\right)$$

(g)  $y = x^3 - 3x^2 - 1$

$$y' = 3x^2 - 6x$$

(h)  $f(x) = 7x - 3$

$$f'(x) = 7$$

(i)  $f(x) = e^\pi$

$$f'(x) = 0$$

(j)  $f(x) = \frac{1}{\sqrt{x}}$

$$f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$$

(k)  $f(x) = 5\sqrt[3]{x} + 6x^2 - e$

$$f'(x) = \frac{5}{3}x^{-\frac{2}{3}} + 12x$$

(l)  $f(x) = \frac{1}{x^2} + \frac{1}{x} + 1$

$$f'(x) = -\frac{2}{x^3} - \frac{1}{x^2}$$

(m)  $f(x) = \sin 3$

$$f'(x) = 0$$

(n)  $f(x) = x^2 \sin 2$

$$f'(x) = (2 \sin 2)x$$

(o)  $f(x) = x^e + x^\pi$

$$f'(x) = ex^{e-1} + \pi x^{\pi-1}$$

2. Compute the following derivatives:

(a)  $\frac{d}{dx}\left(\frac{x^3}{2}\right)$

$$= \frac{3}{2}x^2$$

(b)  $\frac{d}{dx}\left(\frac{3x+2}{2x-11}\right)$

$$= \frac{3(2x-11) - 2(3x+2)}{(2x-11)^2} = \frac{6x-33-6x-4}{(2x-11)^2} = \frac{-37}{(2x-11)^2}$$

(c)  $\frac{d}{dx}(\sqrt{\sin^2 x})$

$$= \frac{d}{dx} \sin x = \cos x$$

(d)  $\frac{d}{dx}((\sin^2(2x) + 1)^e)$

$$\begin{aligned} &= e(\sin^2(2x) + 1)^{e-1}(2 \sin(2x) \cdot \cos(2x) \cdot 2) \\ &= 4e \sin(2x) \cos(2x)(\sin^2(2x) + 1)^{e-1} \end{aligned}$$

(e)  $\frac{d}{dx}(\tan(\cos(\sin x)))$

$$= \sec^2(\cos(\sin x))(-\sin(\sin x))(\cos x)$$

(f)  $\frac{d}{dx}((\tan x)(\cos x)(\sin x))$

$$= \sec^2 x \cos x \sin x - \tan x \sin^2 x + \tan x \cos^2 x$$

(g)  $\frac{d}{dx}((4x+3)^4(x+1)^{-1})$

$$= 4(4x+3)^3 \cdot 4 \cdot (x+1)^{-1} - (x+1)^{-2}(4x+3)^4$$

(h)  $\frac{d}{dx}\left(\sin\left(\frac{3\pi x}{x}\right) + \cos\left(\frac{3\pi x}{2}\right)\right)$

$$= -\sin\left(\frac{3\pi x}{2}\right) \cdot \frac{3\pi}{2}$$

(i)  $\frac{d}{dx}(x \tan x)$

$$= \tan x + x \sec^2 x$$

(j)  $\frac{d}{dx}((x^2+1) \sec x)$

$$= 2x \sec x + (x^2+1) \sec x \tan x$$

$$(k) \frac{d}{dx} (x(x^2 + 1) \tan x \sec x)$$

$$= \frac{d}{dx} ((x^3 + x) \tan x \sec x)$$

$$= (3x^2 + 1) \tan x \sec x + (x^3 + x) \sec^3 x + (x^3 + x) \sec x \tan^2 x$$

$$(l) \frac{d}{dx} (x \sin^{-1} x + \sqrt{-x^2})$$

$$= \sin^{-1} x + \frac{x}{\sqrt{1-x^2}} + \frac{1}{2}(-x^2)^{-\frac{1}{2}}(-2x)$$

$$= \sin^{-1} x + \frac{x}{\sqrt{1-x^2}} + -\frac{x}{\sqrt{-x^2}}$$

$$(m) \frac{d}{dt} (\cot^{-1} \sqrt{t-1})$$

$$= -\frac{1}{1+\sqrt{t-1}^2} \left( \frac{1}{2}(t-1)^{-\frac{1}{2}} \right) = \frac{1}{t} \frac{1}{2} \frac{1}{\sqrt{t-1}}$$

$$= \frac{1}{2t\sqrt{t-1}}$$

$$(n) \frac{d}{ds} (\tan^{-1} \sqrt{x^2-1} + \sin^{-1} x)$$

$$= \frac{1}{1+\sqrt{x^2-1}^2} \frac{1}{2}(x^2-1)^{-\frac{1}{2}} + \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{1}{1+x^2-1} \frac{1}{2\sqrt{x^2-1}} + \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{1}{2x^2\sqrt{x^2-1}} + \frac{1}{\sqrt{1-x^2}}$$

$$(o) \frac{d}{dx} (3^{\sin x})$$

$$= 3^{\sin x} \ln 3 \cos x$$

$$(p) \frac{d}{dx} (\ln(x^2 + 1))$$

$$= \frac{2x}{x^2 + 1}$$

$$(q) \frac{d}{dx} (e^{\sin x})$$

$$= e^{\sin x} \cos x$$

$$(r) \frac{d}{dx} (x^{\sin x})$$

$$y = x^{\sin x}$$

$$\ln y = \sin x \ln x$$

$$\frac{1}{y}y' = \cos x \ln x + \frac{\sin x}{x}$$

$$y' = x^{\sin x} \left( \cos x \ln x + \frac{\sin x}{x} \right)$$

(s)  $\frac{d}{dx} \left( \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}} \right)$

$$y = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}$$

$$\ln y = \ln \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}$$

$$\ln y = \frac{1}{5} \ln \frac{(x-3)^4(x^2+1)}{(2x+5)^3}$$

$$\ln y = \frac{1}{5} (\ln(x-3)^4(x^2+1) - \ln(2x+5)^3)$$

$$\ln y = \frac{1}{5} (\ln(x-3)^4 + \ln(x^2+1) - 3 \ln(2x+5))$$

$$\ln y = \frac{4}{5} \ln(x-3) + \frac{1}{5} \ln(x^2+1) - \frac{3}{5} \ln(2x+5)$$

$$\frac{1}{y}y' = \frac{4}{5} \frac{1}{x-3} + \frac{1}{5} \frac{2x}{x^2+1} - \frac{3}{5} \frac{2}{2x+5}$$

$$y' = y \left( \frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x+5)} \right)$$

$$y' = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}} \left( \frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x+5)} \right)$$

(t)  $\frac{d}{dx} (x \sin x + e^x + \pi^x + x^\pi)$

$$= \sin x + x \cos x + e^x + \pi^x \ln \pi + \pi x^{\pi-1}$$

(u)  $\frac{d}{dx} (\ln 2 \cdot \log_2 x)$

$$= \frac{d}{dx} \left( \ln 2 \cdot \frac{\ln x}{\ln 2} \right)$$

$$= \frac{d}{dx} (\ln x)$$

$$= \frac{1}{x}$$

(v)  $\frac{d}{dx} (\ln 10^x)$

$$= \frac{d}{dx} (x \ln 10)$$

$$= \ln 10$$

$$\begin{aligned}
\text{(w)} \quad \frac{d}{dt} \left( \frac{t^2 + 3t}{t} \right) &= \frac{(2t + 3)t - (t^2 + 3t)}{t^2} \\
&= \frac{2t^2 + 3t - t^2 - 3t}{t^2} \\
&= \frac{t^2}{t^2} \\
&= 1
\end{aligned}$$

$$\begin{aligned}
\text{(x)} \quad \frac{d}{dx} (x \ln x) &= \ln x + 1
\end{aligned}$$

$$\begin{aligned}
\text{(y)} \quad \frac{d}{dx} \left( \frac{e^x}{e^{-x} + 1} \right) &= \frac{e^x(e^{-x} + 1) - (-e^{-x})(e^x)}{(e^{-x} + 1)^2} \\
&= \frac{1 + e^x + 1}{(e^{-x} + 1)^2} \\
&= \frac{2 + e^x}{(e^{-x} + 1)^2}
\end{aligned}$$

3. Find  $\frac{dy}{dx}$  for each of the following;

$$\text{(a)} \quad xy + 2x + 3y = 1$$

$$\begin{aligned}
xy + 3y &= 1 - 2x \\
y(x + 3) &= 1 - 2x \\
y &= \frac{1 - 2x}{x + 3} \\
\frac{dy}{dx} &= \frac{-2(x + 3) - (1 - 2x)}{(x + 3)^2} \\
&= \frac{-2x - 6 - 1 + 2x}{(x + 3)^2} \\
&= \frac{-7}{(x + 3)^2}
\end{aligned}$$

$$\text{(b)} \quad \sqrt{xy} + y^2 = 1$$

$$\begin{aligned}
\frac{1}{2}(xy)^{-\frac{1}{2}} \left( y + \frac{dy}{dx}x \right) + 2y \frac{dy}{dx} &= 0 \\
\frac{1}{2\sqrt{xy}} \left( y + \frac{dy}{dx}x \right) + 2y \frac{dy}{dx} &= 0
\end{aligned}$$

$$\frac{y}{2\sqrt{xy}} + \frac{x}{2\sqrt{xy}} \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$\frac{x}{2\sqrt{xy}} \frac{dy}{dx} + 2y \frac{dy}{dx} = -\frac{y}{2\sqrt{xy}}$$

$$\frac{dy}{dx} \left( \frac{x}{2\sqrt{xy}} + 2y \right) = -\frac{y}{2\sqrt{xy}}$$

$$\frac{dy}{dx} = \frac{-\frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}} + 2y}$$

(c)  $\sin^2 x + \cos^2 y = 1$

$$2 \sin x \cos x - 2 \cos y \sin y \frac{dy}{dx} = 0$$

$$2 \sin x \cos x = 2 \cos y \sin y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2 \sin x \cos x}{2 \cos y \sin y}$$

(d)  $\frac{x^3 - xy}{x} = 3y$

$$x^2 - y = 3y$$

$$x - \frac{dy}{dx} = 3 \frac{dy}{dx}$$

$$2x = 4 \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2}x$$

(e)  $x(t) = 2t^7 - 4t^2 + 5$

$$y(t) = 3t^2 + 5$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy}{dt} = 6t$$

$$\frac{dx}{dt} = 14t^6 - 8t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t}{14t^6 - 8t}$$

$$(f) \quad x(t) = \cos(5t^2 + t)$$

$$y(t) = \cos^2(7t^3 - t^2)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\begin{aligned} \frac{dy}{dt} &= 2 \cos(7t^3 - t^2)(-\sin(7t^3 - t^2)(21t^2 - 2t)) \\ &= -2(21t^2 - 2t) \cos(7t^3 - t^2) \sin(7t^3 - t^2) \end{aligned}$$

$$\frac{dx}{dt} = -\sin(5t^2 + t)(10t + 1)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2(21t^2 - 2t) \cos(7t^3 - t^2) \sin(7t^3 - t^2)}{(10t + 1) \sin(5t^2 + t)}$$