Math 142
Spring, 2005
Worksheet - §P.4

Name: **Solutions**

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1. Simplify the following expressions:

   (a) $\ln e^3$

   $$= 3$$

   (b) $10^{\log 7.4}$

   $$= 7.4$$

   (c) $\log_8 16$

   $$\log_8 16 = y$$
   $$8^y = 16$$
   $$(2^3)^y = 2^4$$
   $$2^{3y} = 2^4$$
   $$3y = 4$$
   $$y = \frac{4}{3}$$
   $$\log_8 16 = \frac{4}{3}$$

   (d) $\log_{25} 5 = \frac{1}{2}$
   since

   $$25^{\frac{1}{2}} = \sqrt{25} = 5$$

   (e) $\log_2 8 =$

   $$\log_2 8 = x \iff 2^x = 8 \iff x = 3$$

   (f) $\log \frac{1}{10} =$

   $$\log \frac{1}{10} = x \iff 10^x = \frac{1}{10} \iff 10^x = 10^{-1} \iff x = -1$$

(g) $\ln e^{2.78} =$

$$\ln e^{2.78} = x \iff e^x = e^{2.78} \iff x = 2.78$$

OR

$$\ln e^{2.78} = 2.78 \ln e \text{ and } \ln e = 1$$

So

$$2.78 \ln e = 2.78$$

(h) $\log_4 64 =$

$$\log_4 64 = x \iff 4^x = 64 \iff 4^x = 4^3 \iff x = 3$$

(i) $\log_3 \dfrac{1}{\sqrt{27}}$

$$\log_3 \frac{1}{\sqrt{27}} \iff 3^x = \frac{1}{\sqrt{27}} = \frac{1}{\sqrt{3^3}} = \frac{1}{3^{\frac{3}{2}}} = 3^{-\frac{3}{2}} \iff x = -\frac{3}{2}$$

2. Write each expression as a single logarithm:

(a) $\log 4k + \log 5k^3$

$$\begin{aligned} &= \log\big((4k)(5k^3)\big) \\ &= \log 20k^4 \end{aligned}$$

(b) $4\ln x - 2\left(\ln x^3 + 4\ln x\right)$

$$\begin{aligned} &= \ln x^4 - 2\ln x^3 - 8\ln x \\ &= \ln x^4 - \ln x^6 - \ln x^8 \\ &= \ln \frac{x^4}{x^6} - \ln x^8 \\ &= \ln \frac{1}{x^2} - \ln x^8 \\ &= \ln\left(\frac{x^{-2}}{x^8}\right) \\ &= \ln x^{-10} \end{aligned}$$

3. Solve each equation:

(a) $2 \cdot e^{2x+1} = 10$

$$\begin{aligned} e^{2x+1} &= 5 \\ 2x + 1 &= \ln 5 \\ 2x &= \ln 5 - 1 \\ x &= \frac{\ln 5 - 1}{2} \end{aligned}$$

(b) $\ln(2x-1)=3$

$$e^3 = 2x - 1$$
$$e^3 + 1 = 2x$$
$$\frac{e^3+1}{2} = x$$

(c) $\ln(m+3) - \ln m = \ln 2$

$$\ln\frac{m+3}{m} = \ln 2$$
$$e^{\ln\frac{m+3}{m}} = e^{\ln 2}$$
$$\frac{m+3}{m} = 2$$
$$m+3 = 2m$$
$$m = 3$$

(d) $2\ln(y+1) = \ln(y^2-1) + \ln 5$

$$\ln(y+1)^2 = \ln(y^2-1) + \ln 5$$
$$\ln(y+1)^2 - \ln(y^2+1) = \ln 5$$
$$\ln\frac{(y+1)^2}{y^2+1} = \ln 5$$
$$e^{\ln\frac{(y+1)^2}{y^2+1}} = e^{\ln 5}$$
$$\frac{(y+1)^2}{y^2+1} = 5$$
$$y^2+2y+1 = 5y^2+5$$
$$0 = 4y^2 - 2y + 4$$
$$0 = 2(2y^2 - y + 2)$$
$$0 = 2y^2 - y + 2$$
$$y = \frac{1 \pm \sqrt{1-4(2)(2)}}{4}$$

Therefore there are no solutions (the number under the radical is negative).

(e) $\log(m+2) = 1$

$$m+2 = 10^1$$
$$m+2 = 10$$
$$m = 8$$

(f) $\log_2(3k-2) = 4$

$$\begin{aligned} 2^4 &= 3k-2 \\ 16 &= 3k-2 \\ 18 &= 3k \\ k &= \frac{18}{3} \end{aligned}$$

(g) $\log_5\left(\frac{5z}{z-2}\right) = 2$

$$\begin{aligned} 5^2 &= \frac{5z}{z-2} \\ 25 &= \frac{5z}{z-2} \\ 25(z-2) &= 5z \\ 25z - 50 &= 5z \\ 20z &= 50 \end{aligned}$$

(h) $\log_2 r + \log_2(r-2) = 3$

$$\begin{aligned} \log_2\left(r(r-2)\right) &= 3 \\ 2^3 &= r(r-2) \\ 8 &= r^2 - 2r \\ r^2 - 2r - 8 &= 0 \\ (r-4)(r+2) &= 0 \\ r = 4 &\text{ or } r = -2 \end{aligned}$$

Notice that $r = -2$ is not in the domain of the original equation, so this cannot be a solution. Therefore the only solution is $r = 4$.

(i) $2^{3x} = \frac{1}{8}$

$$\begin{aligned} 2^{3x} &= 2^{-3} \\ 3x &= -3 \\ x &= -1 \end{aligned}$$

(j) $\left(\frac{9}{16}\right)^x = \frac{3}{4}$

$$\begin{aligned} \left(\left(\frac{3}{4}\right)^2\right)^x &= \frac{3}{4} \\ \left(\frac{3}{4}\right)^{2x} &= \frac{3}{4} \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

(k) $9^{2y-1} = 27^y$

$$\left(3^2\right)^{2y-1} = \left(3^3\right)^y$$
$$3^{2(2y-1)} = 3^{3y}$$
$$2(2y-1) = 3y$$
$$4y - 2 = 3y$$
$$y - 2 = 0$$
$$y = 2$$

(l) $8^p = 19$

$$\ln 8^p = \ln 19$$
$$p \ln 8 = \ln 19$$
$$p = \frac{\ln 19}{\ln 8}$$

(m) $6^{2-m} = 2^{3m+1}$

$$\ln 6^{2-m} = \ln 2^{3m+1}$$
$$(2-m)\ln 6 = (3m+1)\ln 2$$
$$2\ln 6 - m\ln 6 = 3m\ln 2 + \ln 2$$
$$2\ln 6 - \ln 2 = 3m\ln 2 + m\ln 6$$
$$2\ln 6 - \ln 2 = m\left(3\ln 2 + \ln 6\right)$$
$$m = \frac{2\ln 6 - \ln 2}{3\ln 2 + \ln 6}$$

(n) $2 \cdot 15^{-k} = 18$

$$25^{-k} = 9$$
$$\ln 25^{-k} = \ln 9$$
$$-k\ln 25 = \ln 9$$
$$k = \frac{\ln 9}{-\ln 25}$$